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RESILIENT MODULUS OF WYOMING SUBGRADE SOILS

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16. Abstract

A study was conducted to evaluate the use of subgrade resilient modulus for the design of flexible pavements. The objectives were to: (1) investigate the relationship between resilient modulus and R-value, and (2) develop a predictive equation for resilient modulus based on easily measured soil properties. Thirteen fine-grained soils were investigated. Based on the test results, two multiple linear regression models for predicting resilient modulus are proposed. The equations yield resilient modulus as a function of soil type, the state of stress in the pavement subgrade of soil saturation, R-Value, and other soil properties.

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Preface

This report describes a study conducted at the University of Wyoming to evaluate the use of subgrade resilient modulus for the design of flexible pavements. The objectives of this initial study were to: a) investigate the relationship between resilient modulus and R-value, and b) develop a predictive equation for resilient modulus based on easily measured soil properties. Thirteen different fine-grained soils were investigated. Based on the test results, two multiple linear regression models for predicting resilient modulus are proposed.

The principal result of this research is the development of improved equations for predicting the resilient modulus of silty and clayey subgrade soils in Wyoming. These equations yield the resilient modulus as a function of soil type, the state of stress in the pavement subgrade, the degree of soil saturation, R-value, and other soil properties. This study is the first to provide highway engineers in Wyoming with reliable information on the resilient modulus of representative subgrade soils. The information contained herein can be used by highway engineers to make informed decisions on how to incorporate the recommendations of the 1986 AASHTO Guide for Design of Pavement Structures into design methodology for highway pavements.

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CHAPTER 1

INTRODUCTION

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BACKGROUND AND STATEMENT OF THE PROBLEM

Extensive experience has demonstrated that excessive elastic deflections of roadbed soils can cause cracking, rutting, and eventual fatigue failure of asphalt pavements. Resilient modulus, which represents a rational measure of the elastic response of a soil subjected to repeated loading, has been selected by AASHTO (1986) as the definitive material property to characterize roadbed soils for pavement design. The laboratory equipment currently available to perform the resilient modulus test is complex and expensive. In addition, the laboratory procedure currently specified by AASHTO T274 is not easily adaptable to routine daily testing. Thus, there is considerable interest in estimating resilient modulus based upon more easily performed laboratory tests.

State highway departments in the geographic region covered by the Mountain Plains Consortium (MPC) are in the process of reviewing their methods of obtaining soil support values for pavement design. The long-term objective of most states is to adopt the AASHTO recommendation and convert to the use of resilient modulus. Currently, three states in the MPC (Colorado, Minnesota, and Wyoming) use the Hveem Resistance-value, or R-value, to characterize soil support. The remaining states (North Dakota and Utah) use the California Bearing Ratio, or CBR test. Although empirical correlations exist in the published literature for approximating resilient modulus as a function of R-value, CBR, and other soil properties, these were not developed using subgrade soils from this region and no data exist upon which to evaluate the validity of these correlations for use in pavement design in this region.

OBJECTIVES

The principal objectives of this study were to: a) investigate the relationship between resilient modulus and R-value for representative Wyoming soils and b) develop a regression model to predict the resilient modulus of fine-grained soils (clays and silts) based on easily measured soil index properties and specimen characteristics. Specific predictor variables considered in the model building process included:

1) Soil index properties - Atterberg limits, gradation, soil classification, and specific gravity.

2) Specimen characteristics - Moisture content, dry density, and degree of saturation.

An additional objective of this research was to compare the predictive models for resilient modulus developed in this study to other published models.

SCOPE OF STUDY

Chapter 2 describes the development and rationale of the R-value and resilient modulus tests for subgrade soils. Factors that influence the results of each test are reviewed. Published models for estimating resilient modulus also are reviewed and evaluated. Chapter 3 describes the materials, equipment, and procedures used in this study for collecting and testing subgrade soils. The soils consisted essentially of fine-grained soils, representing AASHTO M145 (1986) soil classifications A-4, A-6, and A-7-6. Table 4.5 summarizes the range and statistics of the soil index properties and specimen characteristics. Chapter 4 describes the statistical analysis of the laboratory test results and the model building process. Two final regression equations (equations

4.8 and 4.9) are proposed based on the analysis. Chapter 5 summarizes and concludes this study and suggests several areas for further research.

CHAPTER 2

REVIEW OF PREVIOUS WORK

This chapter describes the historical development, procedures, and application of the two testing methods used in this study to evaluate the soil support characteristics of pavement subgrades. These are: 1) the R-value test and 2) the resilient modulus test using a repeated load triaxial apparatus. The factors that influence the results of each test are reviewed and evaluated. Published correlations between resilient modulus and other soil properties, including R-value, are reviewed and evaluated.

THE R-VALUE TEST FOR SUBGRADE SOILS

Development of the Hveem Stabilometer

Bituminous surfacing was a relatively new process in the United States in the 1920s, and at the time there was a pressing need to evaluate the stability of bituminous paving mixtures. The term stability, as generally used over the last seventy years by highway materials engineers, refers to the ability of a paving material to resist deformation under sustained or repeated traffic loads (Stanton & Hveem 1935, Asphalt Institute 1984).

In the early 1930s, F.N. Hveem, working for the Materials Research Section of the California Division of Highways, considered the problem of measuring stability. He reasoned that if a cylinder of bituminous paving material is confined laterally while being loaded vertically, a very simple relationship must exist between the vertical and horizontal stresses (Endersby 1950). Using this line of reasoning Hveem developed a modified triaxial test to measure the horizontal deformation of a cylindrical specimen under a slowly applied vertical load.

The first device developed by Hveem was relatively crude, consisting of a metal split cylinder connected by a restraining coil spring and an Ames dial gage (Stanton & Hveem 1935). The test specimen was confined in the metal cylinder and, as an axial compression load was applied, the lateral movement or expansion of the metal sleeve was measured. In 1932, this device was replaced by a hydraulic design shown in Figure 2.1, in which lateral stress is transmitted through the walls of a rubber tube to an annular fluid. Lateral stress is measured by means of a standard hydraulic pressure gauge. A screw pump also is provided for adjusting the volume of fluid in the cell.

Stanton and Hveem (1935) established an arbitrary scale on which zero corresponds to a liquid having no measurable internal resistance to a slowly applied vertical load and, at the other extreme, 100 corresponds to a hypothetical solid that does not deform laterally under an axial load and, therefore, does not increase the pressure in the surrounding fluid.

Stability

The process of testing a bituminous paving specimen in the stabilometer essentially consists of subjecting the specimen to stresses sufficient to represent the combined effects of traffic loads frequently repeated over a long period of time (Hveem & Davies 1950). A vertical load is slowly applied to the specimen while the resultant increase in horizontal stress is recorded at 1000 pound increments up to a maximum vertical load of 6000 lbs.

The stability of the bituminous mixture is calculated by the empirically derived formula:

$$S = \frac{22.2}{(P_H * D)/(P_V - P_H) + 0.222} \tag{2.1}$$

where:

S = Stability value (0 - 100);
P_H = Horizontal stress at P_V = 400 psi;
D = Turns displacement;
P_V = Vertical stress (400 psi).

The test is not carried out to yield or failure (Hveem & Davis 1950). Instead, the horizontal stress (P_H) measured at a vertical stress (P_V) of 400 psi represents what was considered by Stanton and Hveem (1935) to be the actual stresses developed by truck traffic in the 1930s.

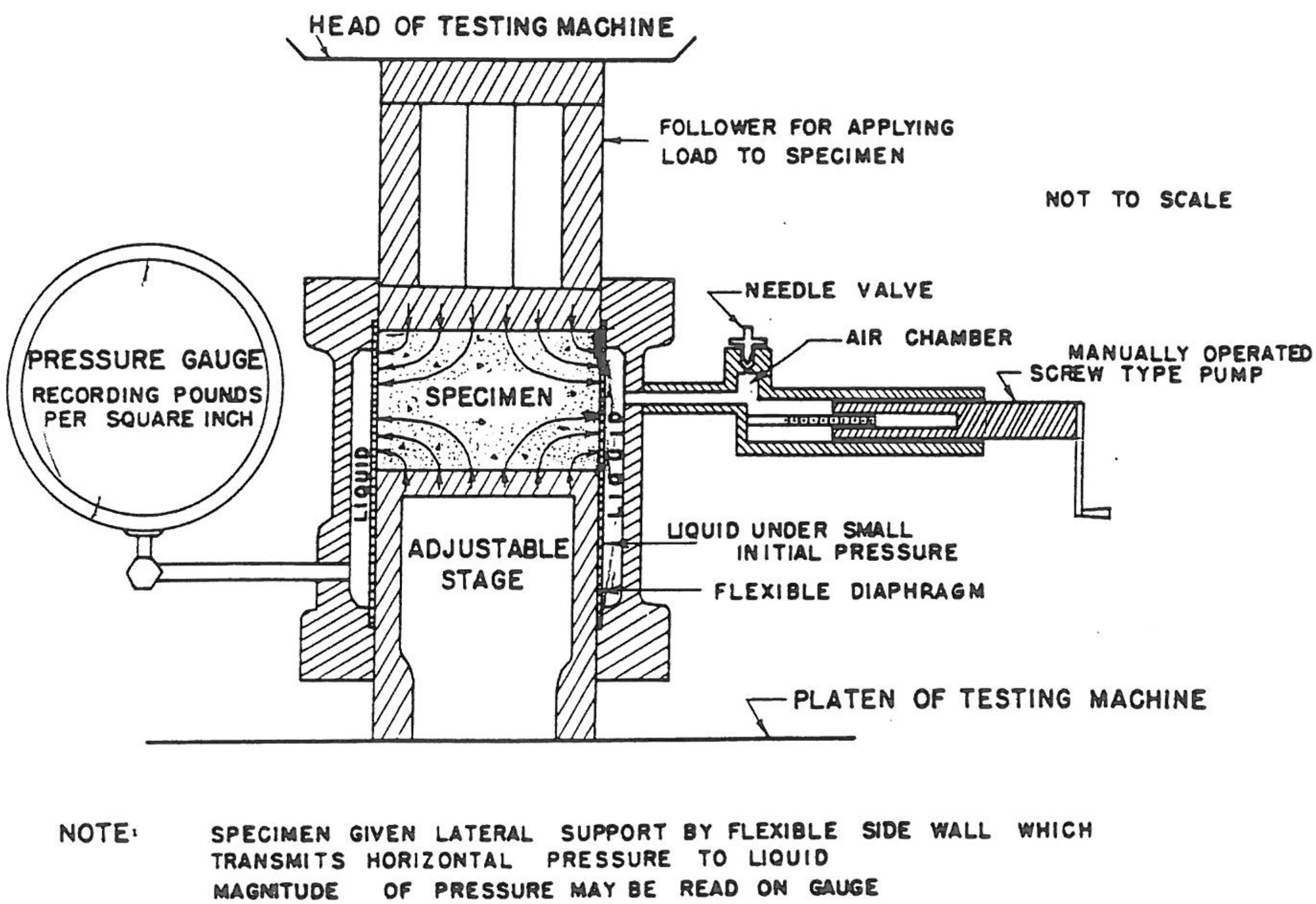

Figure 2.1. Diagrammatic Sketch of the Hydraulic Hveem Stabilometer.

R-Value

Beginning in the late 1940s, the Stabilometer was used to study the soil support characteristics of highway subgrades and bases (Hveem & Carmay 1948). Hveem coined the term "Resistance value" (R-value) to describe the relative tendency of a subgrade material to transmit lateral stress. R-value is computed as follows:

$$R = 100 - \frac{100}{(2.5/D)*(P_V/P_H - 1) + 1} \qquad (2.2)$$

where: R = Resistance value;

P_H = Horizontal stress at P_V = 160 psi;

P_V = Vertical stress (160 psi);

D = Turns displacement.

The horizontal stress is measured at an applied vertical stress of 160 psi (AASHTO T190). According to Hveem (1947), 160 psi was adopted as an average value of the vertical stress developed in a typical pavement subgrade. This arbitrary value of vertical stress (160 psi) does not appear to have a significant influence on the result. Hveem (1947) reported no marked variation in the ratio of P_V/P_H for values of P_V ranging from 100 to 400 psi.

As with stability, R-value is scaled from 0 to 100 with 0 corresponding to a liquid or very plastic clay and 100 corresponding to a theoretical solid for which the application of vertical load results in the transmission of no horizontal pressure in the surrounding fluid (Hveem & Carmay 1948).

Factors Affecting R-Value

Method of Compaction. In the 1940s Hveem designed a kneading-type compactor in an attempt to produce laboratory specimens representative of actual field samples (Hveem & Davis, 1950). The compactor, now known as the California kneading compactor, uses a tamping foot shaped like a segment of a circle and approximately 3 square inches in area. The tamping foot develops a kneading action, simulating a steel drum or rubber-tired pneumatic roller. The sample mold rotates 1/6 turn between tamps so that the foot strikes a different area each time. While the kneading action provides stress applications similar to the effect of field compaction, two important differences have been suggested by Majidzadeh, et al. (1980):

1) The pressure intensity applied by the compactor foot is much higher than that encountered in the field.
2) Local resistance may develop due to sample confinement in a rigid mold.

Exudation Pressure. After being removed from the compactor, the specimen, while still confined in the steel mold, is subjected to a static stress ranging up to 800 psi until moisture exudes from the specimen. The stress at the point where moisture exudes is called the exudation pressure (AASHTO T190). The intent of this procedure is to approach saturation in the specimen by raising the pore water pressure to a level high enough for the water to absorb into solution most of the air originally in the void space. Once the specimen is close to saturation, any further increase in load is carried by the pore fluid and will cause water to flow or exude from the specimen (Hveem & Carmay 1948).

Studies performed by the California Department of Highways in the 1950s indicated that an exudation pressure of 300 psi represented the worst condition likely to be reached in a typical subgrade material in place several years after construction (Howe 1961). The study was based upon overall soil types, climate, drainage, and highway construction conditions existing in California in the 1950s.

Berry and Hines (1983) reported that, for A-7-6 subgrade soils typical to Colorado, the average moisture content of laboratory specimens prepared at a 300 psi exudation pressure was 2.7 percent higher than the average in-situ moisture content. The average moisture content of an A-6 subgrade soil at 300 psi exudation pressure was found to be approximately equal to the average in-situ moisture content.

Surface Roughness. The degree of surface roughness can significantly influence the amount of specimen deformation that occurs, which in turn affects the amount of pressure developed in the stabilometer (Hveem & Davis 1950). The turns displacement compensates for variations in surface roughness. The turns displacement value represents the volume of fluid required to increase the pressure in the apparatus from 5 to 100 psi, expressed as the number of turns or revolutions of the calibrated pump handle (AASHTO T190). A standard initial displacement of two revolutions of the pump handle, when a dummy metal specimen is used, was adopted by Hveem and Davis (1950) for calibrating the stabilometer.

Note that equation 2.2 can be transformed to a much simpler form when the roughness correction is removed:

$$R = (1-P_h/P_v)*100 \tag{2.3}$$

This form of the expression for R-value was proposed originally by Hveem and Carmay in 1948 and is related directly to Hveem's original idea of expressing R-value as a function of the applied vertical stress and the resultant horizontal stress. The roughness correction was added several years later by Hveem and Davis (1950).

RESILIENT MODULUS OF SUBGRADE SOILS

Since the mid-1950s a number of studies have been conducted to evaluate the deformation characteristics of compacted soils subjected to repeated loading. Early efforts to measure resilient modulus in the laboratory by Seed et al. (1955), Seed and McNiel (1958), and Hveem et al. (1962) were prompted, to a large extent, by deflection data collected at the WASHO Road Test (1955) and a comprehensive deflection study described by Hveem (1955) and conducted by the California Department of Highways on roads throughout that state in 1951.

Results of the California study indicated a marked difference in pavement deflections occurring under moving and standing wheel loads (Hveem 1955). In addition, a close correlation between observations of cracking and fatigue failures in bituminous pavements and measured deflections also was found (Hveem 1955).

Laboratory tests performed by Seed et al. (1962) showed a significant difference between the initial tangent moduli determined in conventional triaxial compression tests and moduli determined by tests in which the load is applied repeatedly. Barksdale (1971) showed, based on analysis of layered pavements using linear and nonlinear elastic finite element analysis, that the repeated load triaxial test more closely simulates stress conditions than the Hveem stabilometer, the CBR test, or the conventional triaxial test. These findings, along with the early WASHO and California field studies, suggest that the response of soils under traffic loading can only be simulated and measured accurately in the laboratory by using repeated loading tests.

The early laboratory studies in the 1950s were concerned primarily with fine-grained subgrade soils. Investigation of coarse-grained soils and crushed granular materials followed in the 1960's (Haynes & Yoder 1963, Seed et al. 1967, Monismith et al. 1967).

Definition of Resilient Modulus

The term resilience was introduced by Hveem (1955) for pavement and subgrade materials when he stated, "Resilience is preferred to such terms as elasticity as we are concerned with movements much greater than would be the case in many elastic solids such as glass, concrete, and steel."

Resilient modulus is obtained by subjecting a specimen to repeated applications of vertical stress and measuring the resilient, or recoverable, strain following a specified number of load applications, as illustrated in Figure 2.2. Initially the total strain for one cycle of loading consists of both a resilient portion plus some permanent strain. Following some finite number of cycles, a hysteresis loop will develop, i.e. there will be no further increase in the permanent strain and the specimen will behave elastically over that range of stress.

Resilient modulus is computed by the formula (AASHTO T274):

$$M_R = \frac{\sigma_d}{\varepsilon_R} \tag{2.4}$$

where: σ_d = repeatedly applied deviator stress;

ε_R = recoverable axial strain.

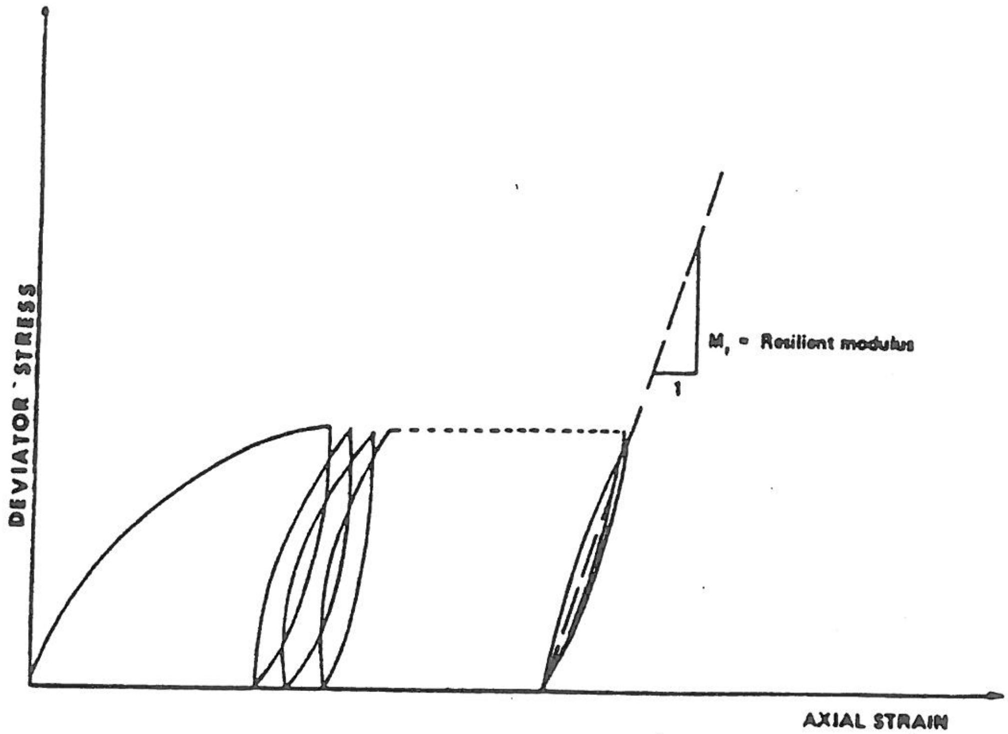

Figure 2.2. Definition of Resilient Modulus.

Factors Influencing the Resilient Modulus of Fine-Grained Soils

State of Stress. Several studies have shown that, for a given set of conditions, i.e. moisture content, density, and soil structure, the resilient modulus of fine-grained soils is principally a function of deviator stress, σ_d (Seed et al. 1962, Fredlund et al. 1972, Thompson and Robnett 1979).

Several models have been proposed to describe this behavior. Thompson and Robnett (1979) proposed a bilinear model determined by linear regression, which is represented by two, usually distinct, sections of a typical resilient modulus versus deviator stress plot. Monismith (1989) proposed a log-linear relationship described by:

$$M_R = A\sigma_d^{\ B} \tag{2.5}$$

where: A and B are regression coefficients.

Number of Load Repetitions. Early research performed by Seed et al. (1962) on subgrade material from the AASHO Road Test indicated that resilient strain, at applied deviator stresses in the range of 1 to 10 psi, increased slightly up to approximately 100 load applications and was essentially stable up to 100,000 load applications. At higher applied deviator stress levels a stiffening effect was noticed at approximately 5000 load applications.

Monismith et al. (1967) reported test results on a natural subgrade material, with an R-value of eight at an exudation pressure of 300 psi, that indicated resilient strain tends to be stable or decreases slightly with repeated loading up to approximately 1000 load applications and then remains constant or increase very slightly. Tests by Townsend and Chisolm (1976) on a heavy clay showed a stiffening effect similar to that observed by Seed et al., but it was more pronounced in samples with a high degree of saturation subjected to higher repetitive loading.

Method of Compaction. The method of compaction plays an important role in the resilient response of fine-grained soils, especially for specimens with substantial amounts of clay (Seed et al. 1962). Compaction data on cohesive soils indicated that compaction in the field at a degree of saturation greater than 80 percent tended to result in a dispersed soil structure, whereas the same soil compacted at a degree of saturation less than 80 percent tended to have a flocculated structure. In general, clays with flocculated structures were found to be more stiff than clays with dispersed structures and thus tended to yield higher resilient moduli.

Thixotropy. In the same study, Seed et al. (1962) tested identical samples for resilient modulus at intervals between compaction and testing ranging from 15 minutes to 50 days. The results indicated that the application of several hundred cycles of loading to the 50 day samples caused a substantial loss of thixotropic stiffness. After 1000 cycles, the resilient strain of the 50-day samples was approximately equal to the one-day samples.

Degree of Saturation. Degree of saturation is a factor that reflects the combined effects of density and moisture content on resilient modulus (Thompson 1989). Thompson and Robnett (1978) reported a strong correlation between resilient modulus and degree of saturation ($r = 0.706$). Their research indicated that as the degree of saturation increases there is a corresponding linear decrease in resilient modulus. Tests performed by Seed et al. (1962) and Jones and Witczak (1972) tend to confirm this behavior.

Other Soil Properties. Thompson and Robnett (1979) have shown that for a given compaction condition, resilient modulus is significantly correlated with liquid limit, plasticity index, group index, clay content, and organic carbon content.

Factors Affecting the Resilient Modulus of Granular Soils

State of Stress. For coarse-grained soils or crushed granular material, the resilient modulus for a given set of conditions is primarily a function of the applied stresses. A number of models have been developed to describe this stress dependency (Monismith

1967, Allen 1973, Uzan 1985). For general highway pavement design purposes, Monismith (1989) recommends:

$$M_R = K_1\theta^{K_2} \tag{2.6}$$

where: θ = first stress invariant ($\theta_1 + \theta_2 + \theta_3$);

K_1 and K_2 = regression coefficients.

By varying the chamber pressure (σ_3) and the deviator stress (σ_d), a series of resilient moduli can be obtained for each sample. The regression coefficients K_1 and K_2 are determined by applying nonlinear regression analysis to the laboratory test results.

Stress History. One of the more important findings, especially from the standpoint of developing a practical laboratory test procedure, is that one specimen may by used to measure the resilient modulus over the entire range of stress levels. Laboratory evidence collected by Allen (1973), Brown and Hyde (1975), and Kallcheff and Hicks (1973) have all indicated that loading history has only a negligible effect on resilient modulus of granular soils.

Duration and Load Frequency. Kallcheff and Hicks (1973) found resilient modulus to be independent of load frequency and load duration in the range of 1 to 3 seconds. Similar findings were reported by Allen and Thompson (1974).

Density. Research by Rada and Witczak (1981) indicated that density is an important predictor variable of resilient modulus, causing an increase in K_1 (equation 2.7). Allen and Thompson (1974) and Hicks and Monismith (1971) have also reported that K_1 increases with increasing density, however, the effect on resilient modulus was not as pronounced as suggested by Rada and Witczak.

Degree of Saturation. Research performed by Rada and Witczak (1981) indicated that the degree of saturation has a significant influence on resilient modulus. Their research suggests that there is a critical degree of saturation near 80 to 85 percent above which granular materials tend to become unstable and deteriorate badly with time.

Gradation. Results presented by Haynes and Yoder (1963) for gravel and crushed rock indicated that the minus No. 200 material had little effect on resilient modulus. The percentage of minus No. 200 in the study ranged from 6 to 11 percent.

REVIEW OF EXISTING EQUATIONS FOR ESTIMATING RESILIENT MODULUS

Several regression equations have been presented in the literature for estimating resilient modulus. Predictor variables typically include moisture content, density, confining stress, soil index properties, and strength properties such as R-value and California Bearing Ratio. The relationships are based upon experimental data for resilient moduli obtained in the laboratory on undisturbed and/or remolded specimens of varying dimensions. In some cases, resilient moduli were determined using linear and nonlinear layered elastic theory on data gathered in the field using a Benkleman Beam, Dynaflect, or Falling Weight Deflectometer.

One of the more commonly used relationships between resilient modulus and other strength tests is the soil support scale illustrated in Figure 2.3. The soil support scale was developed as part of the AASHO Road Test to represent subgrade

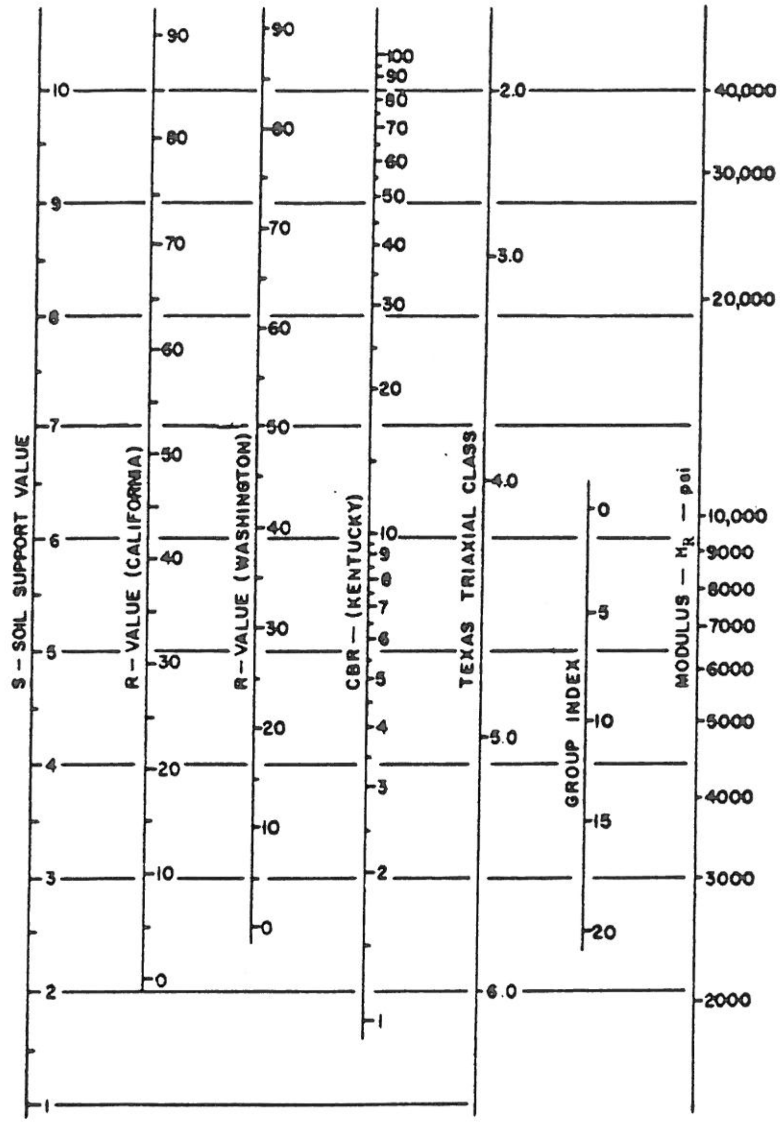

Figure 2.3. The Soil Support Scale.

SOURCE: Van Til et al. (1972).

support (Van Til et al. 1972). A 3.0 on the scale represents the silty clay roadbed soils used on the road test. A 10.0 represents crushed rock base material also used in the road test. The strength scales such as R-value and CBR are based upon data supplied by state highway departments. The resilient modulus scale was developed by applying layered elastic theory to several different subgrade materials (Van Til et al. 1971).

One of the shortcomings of Figure 2.3 is that no distinction is made between fine and coarse-grained soils. The differences in behavior between these two groups of soils makes it important to consider this distinction when evaluating the applicability of prediction equations.

Fine-Grained Soils

Data from the San Diego Test Road project have been used by several researchers to develop regression equations for estimating resilient modulus. The test road was constructed in 1966 and consisted of 35 different sections with varying thicknesses of aggregate and asphalt-bound courses. The subgrade consisted of an A-7-6 clay placed and compacted as uniformly as possible. Based on an analysis of 97 undisturbed field samples, Jones and Witczak (1972) developed the following equation for estimating resilient modulus as a function of the degree of saturation and water content :

$$\text{Log } M_R = -.111*(w\%) + 0.0217*(S\%) + 1.179 \tag{2.7}$$

where: M_R = Resilient Modulus(ksi);

$S\%$ = Degree of Saturation(%);

$w\%$ = Water Content(%)

$r^2 = 0.44$

Test Conditions:

$\sigma_d = 6$ psi

$\sigma_3 = 2$ psi

A relationship between resilient modulus and R-value also was derived from the San Diego test road data and reported by the Asphalt Institute (1982) as:

$$M_R = 772 + 369*(R) \tag{2.8}$$

where: M_R = Resilient Modulus (psi);

R = R-value (0-100).

The Asphalt Institute study (1982) also states that further evaluation of the data led to the following relationship between resilient modulus and R-value that more closely correlates with CBR data collected during the road test:

$$M_R(\text{psi}) = 1155 + 555*(R) \tag{2.9}$$

Kirwan and Snaith (1976) proposed a simple method for estimating resilient modulus by preparing a chart of Relative Compaction (R_C) versus Relative Moisture (R_W) with isolines of resilient modulus. The parameters R_C and R_W are defined as:

$$R_C = \frac{\text{In-situ dry density}}{\text{AASHTO Dry Density}} \tag{2.10}$$

$$R_W = \frac{\text{In-situ moisture content}}{\text{AASHTO Opt. moisture content}} \tag{2.11}$$

A series of charts is required for different soil types in a particular region. Verification of this method has not been documented in the literature.

Tri Buu (1980) reported the following relationship between resilient modulus and R-value for fine-grained Idaho subgrade soils. The soils were classified predominately as ML, which describes a low plasticity silt in the Unified Soil Classification System.

$$M_R(\text{ksi}) = 1.6 + .038*(R) \tag{2.12}$$

Test Conditions: $\sigma_d = 6$ psi

$\sigma_3 = 2$ psi

The reported correlation coefficient was $r = 0.33$. Buu suggested that this low value could be accounted for in part due to poor LVDT sensitivity. He also advised that this relationship is valid only for soils with an R-value higher than 20.

As part of an investigation concerned primarily with correlation of the Colorado Highway Department's R-value on the soil support scale, Hines (1978) presented a graph showing the log of resilient modulus versus R-value. A best-fit analysis of that graph by the first author yields the following equation:

$$\log M_R(\text{ksi}) = 0.0131(\text{R-value}) + .4764 \qquad (2.13)$$

$r = 0.938$

Resilient modulus was determined using linear layered elastic theory to analyze field deflection data collected using a dynaflectometer. Specimens for R-value determination were prepared at moisture contents and densities corresponding to the in-situ conditions at the time of the dynaflect tests. Soil samples tested in the Colorado study were AASHTO M145 Soil types A-2, A-4, A-6, and A-7-6.

Thompson and Robnett (1979) conducted an extensive study of typical fine-grained Illinois subgrade soils and found that the degree of saturation is an important predictor of resilient modulus. The following two equations were reported for predicting resilient modulus as a function of the degree of saturation (S%):

1) AT 95% AASHTO T99 maximum dry density:

$$M_R\ (\text{ksi}) = 32.9 - 0.334^*(S\%) \qquad (2.14)$$

$r = 0.64$

2) At 100% AASHTO T99 maximum dry density:

$$M_R \text{ (ksi)} = 45.2 - 0.428*(S\%) \tag{2.15}$$

r = 0.706

Thompson and Robnett (1979) also developed first-order regression equations in the form of:

$$M_R = a + b_1x_1 + b_2x_2 + \ldots b_nx_n \tag{2.16}$$

where: a and b_i = regression coefficients;

x_i = common soil index properties.

Edris and Lytton (1977) investigated three different soils classified as CH, CL, and ML (Unified Soil Classification System) having clay contents ranging from 20 to 70 percent. Resilient modulus was found to be principally a function of deviator stress, soil suction, clay content, and temperature.

Carmichael and Stuart (1985), based on an extensive literature survey, derived the following regression equation using previously reported resilient modulus test data:

$$\begin{aligned} M_R = {} & 37.341 - 0.456*(PI) - 0.618*(\%W) - 0.1424*(S200) + 0.1791*(CS) \\ & - 0.3248*(DS) + 36.422*(CH) + 17.097*(MH) \end{aligned} \tag{2.17}$$

where: PI = plasticity index;

%W = moisture content;

S200 = percentage passing No. 200 sieve;

CS = confining stress (psi);

DS = deviator stress (psi);

CH = 1 for CH soil;
= 0 otherwise;

MH = 1 for MH soil;
= 0 otherwise.

$r^2 = 0.759$

Coarse-Grained Soils

Rada and Witczak (1981) performed a comprehensive evaluation of resilient modulus using data obtained from 10 different research agencies. Estimates of resilient modulus based on both AASHTO and USCS soil classification systems were found to yield poor results. Typical values of K_1 and K_2 (see Equation 2.6) are summarized in Table 2.1 for the six different categories of granular soils that were evaluated. Resilient modulus for granular materials was found to be influenced primarily by stress state, degree of saturation, and degree of compaction.

Carmichael and Stuart (1985) reported the following regression equation for coarse grained soils:

$$\log M_R = 0.523 - 0.0225*(\%w) + 0.544*(\log T) + 0.173*(SM) + 0.197*(GR) \qquad (2.18)$$

where: M_R = Resilient modulus (psi);

SM = 1 for SM soils (0 otherwise);

GR = 1 for gravel soils (GM, GW, GC, or GP), 0 otherwise;

TABLE 2.1. Summary of K_1 and K_2 Values

Soil Description	K_1	K_2
Silty sands	1620	.62
Sandy gravel	4480	.53
Sand-aggregate	4350	.59
Crushed stone	7210	.45
Limerock	14030	.40
Slag	24250	.37
All data	9240	.52

SOURCE: Rada and Witczak (1981).

T = bulk stress;

%w = moisture content.

r^2 = .836

Comments on Estimating Resilient Modulus

There are numerous equations reported in the literature for estimating resilient modulus for both fine and coarse-grained soils. Careful consideration of the: a) experimental design, b) soil type, c) methods and procedures used, and d) thoroughness of the statistical analysis, is recommended before applying any particular equation to pavement design. Caution also is recommended in attempting to use a particular equation with data that are not within the scope of the original data used to develop the equation.

Finally, it should be noted that since the resilient modulus for both fine and coarse-grained soils is stress dependent, there cannot be a unique relationship between resilient modulus and predictor variables such as R-value, CBR, and soil index properties. A review of the assumptions underlying a particular equation, especially regarding the applied deviator and confining stresses, is an important first step before attempting to apply an equation to pavement design.

CHAPTER 3

EQUIPMENT AND PROCEDURES

This chapter describes the equipment and procedures used in collecting and testing subgrade soil samples for R-value, resilient modulus, index properties, and soil classification.

SAMPLE COLLECTION

The soils used in this research were collected during routine subsurface investigations by the Geology Section of the Wyoming Highway Department (WHD). A broad range of fine-grained soils were collected representing AASHTO M145 soil classifications A-4, A-6, and A-7-6. Samples were documented by project number, station, and vertical limits.

Soil classification and standard R-value (AASHTO T190) tests were performed by the WHD Materials Lab. All other tests for this research were performed by the first author at the University of Wyoming. Table 3.1 lists the soil index and engineering properties determined and the test procedures employed.

SAMPLE PREPARATION AND COMPACTION

The preparation of specimens for the resilient modulus test is based on the view that the density, moisture content, and soil structure should resemble as closely as possible the expected in-service conditions. Since the in-service conditions are difficult to estimate and tend to fluctuate with time and/or seasonal variation, specimens are often prepared over a range of moisture contents and densities.

TABLE 3.1. Soil Engineering Properties and Procedures

PROPERTY	PROCEDURE
Grain Size Analysis	AASHTO T88
Specific Gravity	AASHTO T100
Atterberg limits	
Liquid limit	AASHTO T89
Plastic limit	AASHTO T90
Optimum moisture content (percent by weight)	AASHTO T99
Maximum dry density (pcf)	AASHTO T99
Standard R-value (arbitrary scale 0-100)	AASHTO T190
Modified R-value (arbitrary scale 0-100)	*
Resilient Modulus (psi)	AASHTO T274

*The exudation and swell test phases of AASHTO T190 are omitted. Compaction of specimens is performed using a Hveem kneading compactor but specimen moisture contents and densities are based on a percentage of AASHTO T99 optimum moisture content and maximum dry density, as described in the text.

The Asphalt Institute (1988) recommends using subgrade soil compaction requirements as a guide for preparing M_R specimens. The soil compaction requirements for highway subgrade construction in Wyoming (1987) require a moisture content between -4 and +2 percent of AASHTO T99 optimum moisture content, and a minimum of 95% AASHTO T99 maximum dry density.

The compaction method for M_R specimen preparation, as specified by AASHTO T274 for fine-grained soils, is based on the expected in-service soil structure. The method, which is especially critical for soils containing significant amounts of clay, is selected in accordance with anticipated field compaction and in-service conditions, primarily with respect to degree of saturation. For further discussion see Seed et al. (1962).

Preparation of specimens for the standard R-value test is based on a "worst case" approach. Samples are fabricated with the California kneading compactor. Several specimens are prepared at moisture contents and densities corresponding to exudation pressures in the range of 100 to 800 psi. Exudation subjects the soil to a very high static stress. The controlling exudation pressure of 300 psi, as specified by AASHTO T190, models the worst in-service conditions anticipated (Howe 1961). After the exudation test, specimens are placed in a swell test apparatus and allowed access to water for 24 hours. The amount of water absorbed by the specimen is generally not measured but it is assumed that the sample moisture content corresponds to the "worst" in-service conditions.

In determining which method(s) to employ in preparing M_R specimens, it was considered important to minimize the differences between M_R and R-value specimens with respect to moisture content, density, and soil structure. The effect of large differences between M_R and R-value specimens would be to attenuate predictor models. Therefore, resilient modulus and R-value specimens were prepared at several different moisture contents and densities, as listed in Table 3.2.

TABLE 3.2. Preparation of Resilient Modulus and R-value Specimens

Moisture Content and Density	No. of Resilient Modulus (M_R) Specimens	No. of R-value (R_{vm}) Specimens
$w_1\gamma_1$[a]	1	2[d]
$w_2\gamma_2$[b]	1	2
$w_3\gamma_3$[c]	1	2

[a]$w_1\gamma_1$--Moisture content and density equivalent to R-value specimens prepared at 300 psi exudation pressure.

[b]$w_2\gamma_2$--AASHTO T99 moisture content and dry density at approximately -1 percent of optimum.

[c]$w_3\gamma_3$--AASHTO T99 moisture content and dry density at approximately +2 percent of optimum.

[d]3-4 Standard R-value (R_{vs}) specimens were prepared by the WHD Materials Lab to determine an R-value at an exudation pressure of 300 psi. Moisture content and density of the R_{vs} specimens were determined prior to placement in the stabilometer. Two R_{vm} specimens were then prepared by the UW lab at moisture contents and densities corresponding to an exudation pressure of 300 psi.

Definition of Terms

The symbol R_{vs} is used throughout this report to indicate specimens prepared using standard R-value test procedures in accordance with AASHTO T190. R_{vm} is used to represent R-value specimens prepared at nonstandard moisture contents and densities, as described in Table 3.2.

Specimen Preparation

Standard R-value (R_{vs}) specimens were prepared in accordance with AASHTO T190. Modified R-value (R_{vm}) and resilient modulus (M_R) specimens were prepared by sieving an adequate quantity of pulverized soil over a 3/4 inch sieve and discarding the material retained. A predetermined amount of water was then added to the minus 3/4 inch material to produce one M_R and two R_{vm} specimens. To obtain a uniform moisture distribution, samples were then placed in a plastic bag and stored overnight.

Compaction

R_{vs} specimens were compacted in accordance with AASHTO T190. M_R and R_{vm} specimens were compacted using a mold with an inside diameter of 4.004 inches and a height of 8.5 inches. R_{vm} specimens were compacted to a height of 2.5 inches and M_R specimens to a height of 8.0 inches. Compaction was accomplished using an electro-hydraulic kneading compactor, Model CN-425A, manufactured by Soiltest, Inc.

R_{vm} specimens were compacted in 2 lifts and M_R specimens in 4 lifts. Each lift was scored before the next lift was placed. Trial and error procedures were used to determine compactor foot pressure and number of blows necessary to reach the desired density. Immediately after compaction, R_{vm} and M_R specimens were extruded and trimmed.

A rubber membrane was placed in a 5-inch diameter plastic tube and then a vacuum was applied to the tube. Plastic end caps were placed on the M_R specimen and the tube placed over the specimen. The vacuum was then released allowing the membrane to enclose the specimen. Rubber O-rings were used to seal the membrane to the plastic end caps.

R_{vm} specimens were wrapped carefully in Saran wrap and sealed with tape. Both the M_R and R_{vm} specimens were stored in a moisture room for a minimum of 24 hours to allow dissipation of transient pore pressures developed during compaction and to achieve a uniform moisture distribution.

EQUIPMENT AND PROCEDURES FOR MODIFIED R-VALUE

After the 24 hour curing period, R_{vm} specimen diameter, height, and total weight were recorded. The specimen was then placed in a Hveem stabilometer, Model AP490, manufactured by Soiltest, Inc. R-value was then determined in accordance with AASHTO T190. The details of this procedure are given in the AASHTO specification and will not be repeated here. Further details regarding the loading equipment at the University of Wyoming are given by Farrar (1990). Following determination of R-value, the moisture content was determined by measuring the moist and dry weight of the entire specimen.

Calibration of the stabilometer to the WHD stabilometer was accomplished by periodically comparing R-value test results from a solid polyurethane calibration cylinder 4.0 inches in diameter and 2.5 inches in height.

EQUIPMENT AND PROCEDURES FOR RESILIENT MODULUS

The procedures and equipment used for resilient modulus tests conformed to the guidelines set forth by AASHTO T274. The testing equipment consisted of a repeated-loading triaxial cell, an Instron servohydraulic closed-loop loading system, and a computer-controlled data acquisition system. The triaxial cell was manufactured by Research Associates, Inc., and is similar to the cell illustrated schematically in Figure 3.1.

A repeated vertical load was applied to the specimen by the loading ram at 1.3 second intervals, with a load duration of 0.1 to 0.2 seconds. Confining stress was applied using air pressure. The number of stress pulse applications was monitored by a counter on the Instron loading apparatus.

The actual load applied to the specimen was measured by a 600-pound load cell mounted directly on top of the sample inside of the triaxial chamber. Two linear variable differential transducers (LVDT's) mounted on opposite sides of the sample

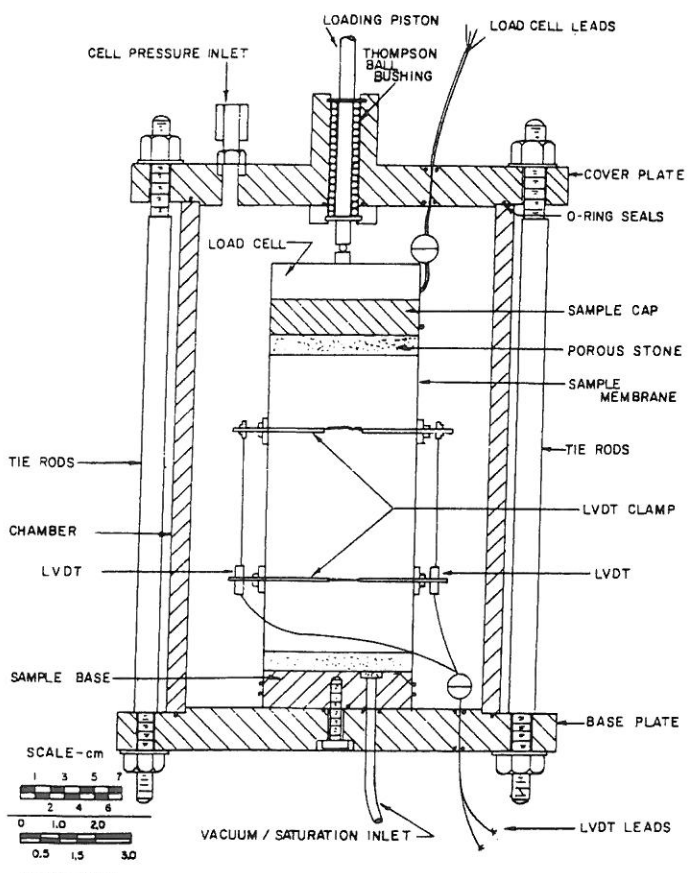

Figure 3.1. Triaxial Cell for Resilient Modulus Tests.

SOURCE: Asphalt institute (1988).

were used to measure the axial deformation. The signals from the two LVDT's were averaged before being recorded. A Keithley data acquisition system controlled by a personal computer was used to obtain the electrical output signal from the load cell and LVDTs, convert them to a digital signal, and record the signal on a computer disc. The computer was also used to convert the load cell signal to stress, and the minimum and maximum stress applied to the specimen for each stress pulse were continuously displayed on the computer screen during the test. The sampling interval was typically in the range of 10 msec. Figures 3.2 and 3.3 show typical curves of deviator stress versus time and sample deformation versus time.

As specified by AASHTO T274, the resilient modulus is determined from the stress-strain curve corresponding to the 200th loading cycle. For this study, data were stored only for the last five loading cycles (cycles 196 to 200). Following the test, the stored data were retrieved. Resilient modulus was determined by analyzing the slope of the stress-strain curve on the final cycle, as defined in Figure 2.2.

Sources of Experimental Error

It was noticed towards the end of the research that the LVDT output sensitivity was more affected by the local and global gain settings on the Keithley Data Acquisition System than originally believed. Determining resilient modulus at deviator stresses of approximately 2 psi or less for relatively stiff specimens requires very careful attention to the amplification of the LVDT signal output. It was determined that, in some cases, the signal amplification was too low, reducing the LVDT sensitivity. After reviewing the data, the investigators chose to not include the test results for resilient modulus determined at deviator stress levels of 1 and 2 psi.

For soft specimens, it was difficult to adjust the Instron controls to force the hydraulic piston to respond at a 0.1 second load duration with the desired load application, even at the maximum gain setting. The actual load duration for these samples varied and was in some cases in the range of 0.2 to 0.3 seconds. The increased load duration may have contributed to excessive plastic deformation (defined as axial strain greater than 4%) observed on several occasions, at deviator stresses of 8 to 10 psi, for specimens of silt prepared at +2 percent of optimum moisture content. On those occasions, the test was stopped and a lower bound of 1000 psi for the resilient modulus was assumed. A value of 1000 psi is the lowest design value in the 1986 AASHTO Design Guide for Pavement Structures and represents a very poor or potentially failed subgrade condition.

Difficulty was sometimes encountered when compacting soil specimens to AASHTO T99 moisture content and density using the kneading compactor. The number of blows and foot pressure required varied considerably between specimens. In future studies comparing resilient modulus and R-value, the following alternate method is suggested: after performing the resilient modulus test, simply section the specimen into two R-value specimens, thus eliminating variation between M_R and R-value specimens with respect to moisture content, density, and soil structure.

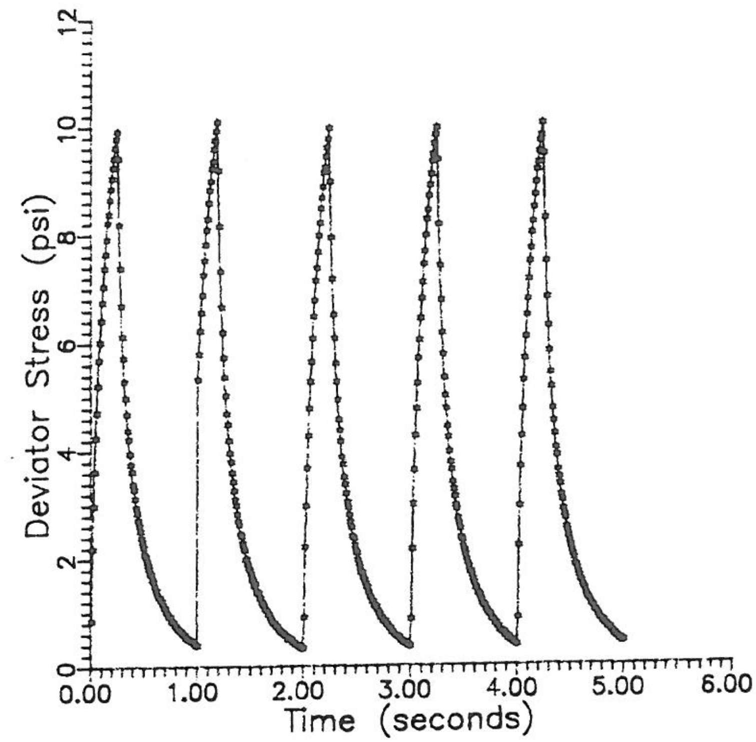

Figure 3.2. Deviator Stress, Measured by 600 Lb. Load Cell, Versus Time.

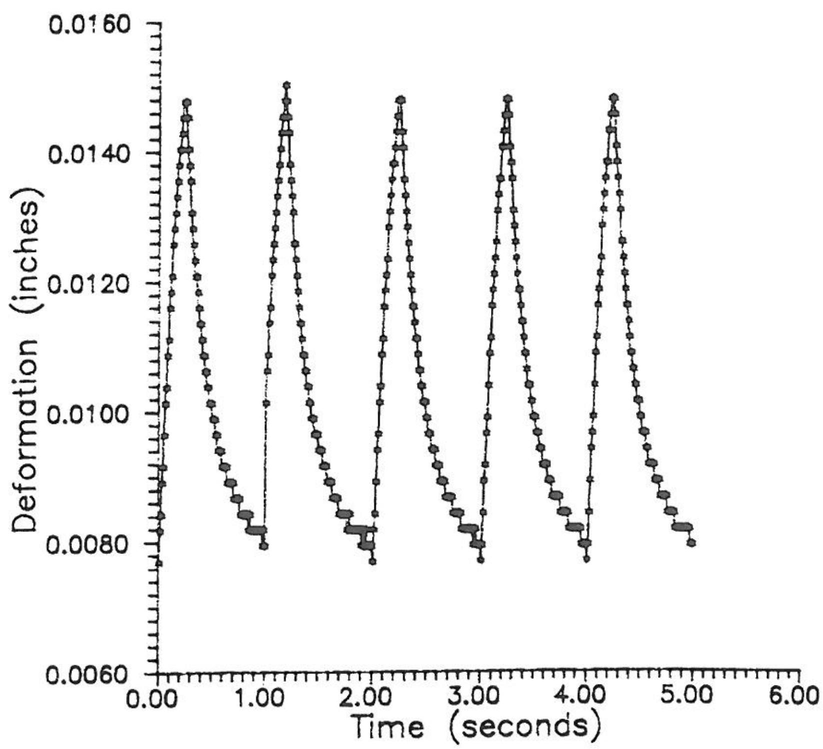

Figure 3.3. Sample Deformation Measured by LVDTs Versus Time.

CHAPTER 4

TEST RESULTS AND ANALYSIS

A total of 29 resilient modulus (M_R) tests, 29 modified R-value (R_{vm}) tests, and 13 standard R-value (R_{vs}) tests were performed on 13 soil samples collected from 4 Wyoming Highway Department (WHD) project sites. Table 4.1 summarizes sample locations, general soil index properties, and AASHTO T99 moisture content and density data. Resilient modulus, standard R-value, and modified R-value test results are presented and discussed.

The results were analyzed statistically using "Statistical Analysis System" software (SAS Users Guide, 1988) on a VAX/VMS mainframe computer. Based on the analysis, equations for estimating resilient modulus are presented, evaluated, and compared to the previously published prediction models reviewed in Chapter 2.

DEFINITION OF REGRESSION TERMS

The purpose of this section is to introduce some common statistical and regression terms used throughout the rest of this chapter.

Simple regression describes the statistical relationship between a response variable (Y) and a predictor variable (X). The response variable (Y) is random and varies with the predictor or independent variable in some systematic fashion. The predictor variable (X) is an independent variable chosen by the investigator. Quantitative predictor variables take on numerical values. Qualitative predictor variables are concerned with non-numerical traits and are called attributes.

Multiple regression is an extension of simple regression in which multiple independent variables are considered. The normal regression model for p-1 independent variables can be expressed as:

$$Y_i = B_o + B_1X_{i,1} + B_2X_{i,2} + \dots + B_{p-1}X_{i,p-1} + \varepsilon_i \tag{4-1}$$

where: B_o, and $B_1,\dots,B_{p-1}$ = regression parameters, and

$X_{i,1},\dots,X_{i,p-1}$ = values of the independent variable on the ith trial.

The error term (ε) is usually assumed to be normally distributed, with a constant variance σ^2 for all observations of x values, and represents the vertical deviation of Y_i from the unknown true regression plane. The regression model is called first-order when the regression parameters and independent variables are linear. Interaction terms are paired combinations of independent variables whose effect on the dependent variable interact. Nonlinear terms usually refer to a transformation of the independent variable, e.g. log(x), sin(x) or x^2.

The variance around the regression plane can be estimated as $(\Sigma(Y - \hat{Y})^2)/(n\text{-}p\text{-}1)$, where $\hat{Y}$ is the fitted regression plane at some vector of x's, Y an observation, n the number of observations, and p-1 the number of independent variables. The residual term e represents the value of $(Y - \hat{Y})$ and is an estimate of the error term. Plots of residuals versus predictor variables and residuals versus predicted values are useful in diagnosing nonconstant variance, nonlinearity, and outliers (extreme values).

The standard error (S_e) of the regression plane is the square root of the estimated variance defined above. S_e estimates the standard deviation of the Y distribution to each possible set of X values. In effect, it measures the dispersion of the data points around the regression plane.

TABLE 4.1. Sample Locations, Soil Index Properties, Optimum Moisture Content And Maximum Dry Density

Sample No.	Project No. And Location	WHD Sample No.	Plasticity Index	Specific Gravity of Solids	Percent Passing 200 Screen	AASHTO Class	AASHTO Max. Dry Density	AASHTO Opt. W%
1.	Colony Rd-Montana So. Preb-048-1(10)	CR(14+16)	38	2.70	89	A-7-6(20)	98.5	21.8
2.	"	CR(6+31)	31	2.78	74	A-7-6(17)	105.3	19.3
3.	Missouri Valley Rd. Preb-0707(1)	MV(6)	17	2.74	82	A-6(11)	109.0	18.2
4.	"	MV(4)	9	2.62	73	A-4(8)	112.5	13.5
5.	Thermopolis-Meeteetse	TM(7+16)	37	2.64	67	A-7-6(15)	104.0	18.0
6.	"	TM(4+6)	16	2.63	83	A-6(10)	108.0	17.0
7.	"	TM(14+15)	18	2.72	82	A-6(10)	117.1	12.8
8.	"	TM(31+32)	20	2.65	77	A-6(12)	110.1	15.5
9.	"	TM(8+9)	5	2.70	47	A-4(2)	119.1	11.7
10.	"	TM(17+18)	3	2.67	43	A-4(2)	118.0	12.0
11.	"	TM(34+38)	4	2.62	50	A-4(3)	117.0	11.8
12.	ALF I-90 Core	ALF(I90C)	21	2.75	85	A-7-6(12)	---	---
13.	ALF I-90 Pit#1	ALF(I90P)	18	2.73	88	A-7-6(11)	111.0	17.7

The coefficient of determination (r^2) is the proportion of the variability of the Y values which can be explained by the model relationship with its x values. For example, if $r^2 = 0.87$, then 87 percent of the variation of the Y values from their mean can be related to the variation in the X values. Expressed mathematically:

$$r^2 = \text{(explained variation of Y) / (total variation of Y).} \qquad (4\text{-}2)$$

The value of r^2 ranges from $0 < r^2 < 1$. The inclusion of an irrelevant variable in a model will increase r^2. The increase occurs because a small part of the fluctuation of the observed variable will, by chance, be explained by the irrelevant variable. To correct for this, r^2 is reduced by a small amount based on the number of observations and independent variables. After the correction, r^2 is usually referred to as the adjusted r^2.

Statistical hypothesis tests (H_o, H_a) refer to an assumption about which of two statements about a parameter should be supported, and are based on information obtained prior to the experiment. In regression analysis, a common hypothesis for the regression parameters is:

$$H_o\colon B = 0,\ H_a\colon B \neq 0 \qquad (4.3)$$

where: H_o = the null hypothesis and

H_a = the alternate hypothesis.

The p-value is defined as the probability of obtaining a value of the test statistic which is equal to or more extreme than the value observed, computed assuming that the null hypothesis is true. If the p-value for the test statistic falls below some specified level of significance (α), then H_o is rejected.

When testing a hypothesis, an experimenter may make the correct decision or commit one of two errors: a) reject the null hypothesis when it is true (referred to as a Type I error), or b) accept the null hypothesis when it is false (referred to as a Type 2 error). The level of significance (α) is the probability of a Type I error occurring. Traditional choices for α are .05, .01, and .001, but an experimenter can choose any value deemed appropriate.

RESULTS OF R-VALUE AND RESILIENT MODULUS TESTS

Standard R-value (R_{vs}) tests were performed by the WHD Materials Lab and are summarized in Table 4-2. All of the R_{vs} values for the clay soil samples (A-7-6, A-6), with the exception of the core sample (Alf I90C), were reported as -5, indicating these soils were so soft that material extruded from under the mold and around the follower ram during the exudation phase of the standard R-value test. R_{vs} results for the silt soils (A-4) are based upon completion of all phases of the standard R-value test. Specimen moisture contents and densities were measured for 3 of the 4 silt soil samples prior to placement in the stabilometer. Three M_R and R_{vm} tests then were performed on specimens prepared at water contents and densities corresponding to an exudation pressure of 300.

Modified R-value (R_{vm}) test results are summarized in Table 4.3, including target moisture contents and densities. Resilient modulus test results are given in Appendix A. For each specimen, nine values of resilient modulus were obtained, corresponding to three values of deviator stress at each of three values of confining stress, as specified by AASHTO T274. The target densities and moisture contents for the resilient modulus specimens are described in Chapter 3. The mean dry density of the M_R specimens, expressed as a percentage of AASHTO T99 maximum dry density, was 100.3 percent.

The relationship between M_R and R_{vs} test results was not investigated since a majority of the standard R-value tests were terminated prior to the stabilometer test phase, when it was observed that soil was being extruded during the exudation test. M_R and R_{vm} test results for specimens prepared at moisture contents and dry densities corresponding to approximately 300 psi exudation pressure were included in the regression analysis.

TABLE 4.2. Standard R-Value (R_{vs}) Test Results

Sample Number	WHD Sample Number	Standard R-Value (R_{vs})
1	CR(14+16)	-5
2	CR(6+31)	-5
3	MV(6)	-5
4	MV(4)	18
5	TM(7+16)	-5
6	TM(4+6)	-5
7	TM(14+15)	-5
8	TM(31+32)	-5
9	TM(8+9)	13
10	TM(17+18)	17
11	TM(34+38)	23
12	ALF(I90P)	-5
13	ALF(I90C)	--

DEVELOPMENT OF PREDICTION MODELS

Soil Type

The data base includes essentially two types of soils: 1) clayey soils (A-7-6 and A-6); and 2) silty soils (A-4). Table 4.4 presents the results of a one-tailed t test comparing the means of the measured response variable (resilient modulus) of the two groups. The hypotheses being tested in this case are:

$$H_o: \mu_c = \mu_s$$

$$H_a: \mu_c > \mu_s$$

where: μ_c = mean resilient modulus for the clay soils;

μ_s = mean resilient modulus for the silt soils.

TABLE 4.3. Modified R-Value (R_{vm}) Test Results

Sample No.	WHD Sample No.	Target Moisture Content and Density[a]	Modified R-Value (Rvm)
1	CR(14+16)	-1	27
2	CR(14+16)	+2	22
3	CR(6+31)	-1	[c]
4	CR(6+31)	+2	14
5	MV(6)	-1	50
6	MV(6)	+2	23
7	MV(4)	-1	59
8	MV(4)	+2	52
9	MV(4)	[b]	59
10	TM(7+16)	-1	23
11	TM(7+16)	+2	18
12	TM(4+6)	-1	17
13	TM(4+6)	+2	13
14	TM(14+15)	-1	46
15	TM(14+15)	+2	32
16	TM(31+32)	-1	33
17	TM(31+32)	+2	17
18	TM(8+9)	-1	31
19	TM(8+9)	+2	9
20	TM(8+9)	300	20
21	TM(17+18)	-1	60
22	TM(17+18)	+2	22
23	TM(17+18)	300	30
24	TM(34+38)	-1	64
25	TM(34+38)	+2	26
26	TM(34+38)	300	27
27	ALF(I90C)	---	10
28	ALF(I90P)	-1	35
29	ALF(I90P)	+2	15

[a] -1 = AASHTO T99 moisture content and density at approximately -1% optimum.
+2 = AASHTO T99 moisture content and density at approximately +2% optimum.
300 = Moisture content and density equivalent to standard R-value specimens prepared at approximately 300 psi exudation pressure.

[b] Moisture content prior to standard R-value Stabilometer test was not recorded, specimen moisture content and density at approximately -1 percent.

[c] Test error.

TABLE 4.4. One-Tailed T Test For Comparing The Mean Resilient Modulus[a] of Two Soil Types

Soil Type	Number of Observations	Mean	Standard Deviation	Standard Error	T	p-value
Silt	101	6,530	5,076	505.1		
Clay	143	8,773	5,054	422.6	1.7	.001

[a] Resilient modulus in pounds per square inch.

The small p-value in Table 4.4 indicates that the null hypothesis should be rejected, i.e. the mean resilient modulus of the clay soil samples was greater than the mean resilient modulus of the silt soil samples at a level of confidence α =.001.

Selection of Predictor Variables

The predictor variables initially considered and their statistics are shown in Table 4.5. The predictor variables, other than R_{vm}, represent relatively easily measured soil index properties, specimen characteristics, or test conditions.

Model Formulation

Figures 1 to 7, presented in Appendix B, are 3-dimensional scatter plots of resilient modulus (M_R), deviator stress (σ_d), and all of the potential predictor variables listed in Table 4.5. Scatter plots are "pictures" of the observed data showing the combined variation of the observed Y values versus the measured predictor variable, X. Scatter plots are useful tools for evaluating whether the relationship between two variables is linear or nonlinear. Figures 8 to 13 in Appendix B are scatter plots of M_R versus the predictor variables for constant values of deviator stress (σ_d = 4 psi) and confining stress (σ_3 = 0 psi).

TABLE 4.5. Variable Statistics

No.	Response Variable	Range	Mean	Std. Dev.
Y	M_R (Resilient Modulus, psi)	1000-20200	7845	5172
	PREDICTOR VARIABLES			
X1	R_{vm} (Modified R-value)	9-64	30.5	16.24
X2	σ_d (Deviator Stress, psi)	3.4-10.55	7.4	2.50
X3	σ_3 (Confining Stress, psi)	0-6	3.0	2.45
X4	w% (Water Content, % by weight)	10.4-23.0	15.56	3.24
X5	γ_d (Dry Density, pcf)	99.7-125.0	111.27	6.23
X6	S% (Percent Saturation(%)	60.4-95.2	81.95	9.77
X7	PI (Plasticity Index)	3-38	16.34	11.49
X8	S200 (% passing 200 screen)	43-89	69.24	16.26
X9	X_c [X_c = 1 for clay soils (A-7-6 and A-6), and X_c = 0 otherwise]			

Inspection of the scatter plots did not reveal any pronounced nonlinear trends. The scatter plots of M_R versus the predictor variables at constant values of deviator and confining stress confirmed that the resilient response of the clay soils is significantly different than the silt soils.

As a starting point, and since nonlinear trends were not pronounced in the scatter plots, a tentative first-order model was examined. All of the variables listed in Table 4.5 were included in the model. The resulting regression model yields a relatively high coefficient of determination, (r^2 = 0.692), but there is considerable multicollinearity among the variables.

Multicollinearity. Multicollinearity is the linear relationship between predictor variables. Multicollinearity does not prevent fitting the observed data, but it inflates the standard errors of the estimated regression coefficients, resulting in questionable p-values for hypothesis tests concerning the regression coefficients (Neter et al. 1989).

Table 4.6 gives the variance inflation factors (VIF's) for the first-order regression model based on the predictor variables listed in Table 4.5. VIF's measure the increase of the variance of the estimated regression coefficients relative to what the variance would be if no multicollinearity existed. When a predictor variable is uncorrelated with the other predictors, its VIF is 1.0. If it is correlated with one or more of the predictors, its VIF is >1.0. A value in excess of 10 is often taken as an indicator that multicollinearity may be unduly inflating the least square estimators (Neter et al. 1989).

From Table 4.6 it can be observed that moisture content (w%), dry density (γ_d), and degree of saturation (S%) have relatively high multicollinearity. Table 4.7 gives the VIF's after removal of w% and γ_d from the model. The coefficient of determination (r^2) of the model without w% and γ_d was essentially unchanged at 0.689 , as compared to r^2 = 0.692 with w% and γ_d in the model.

Bivariate Correlations. Bivariate correlation refers to a linear association between two variables. Table 4.8 presents the correlation matrix for the entire data set based on Pearson's product-moment correlation coefficient. Pearson's product-moment coefficients represent the linear association between any pairs of variables. Typically, a value of r between two predictor variables in excess of 0.8 is considered a problem (Berry, 1985, p. 43).

There is considerable bivariate collinearity among a number of different variable pairs. R_{vm} and S% are highly negatively associated (r = -.861) and individually explain the largest amount of variation in M_R. The qualitative variable X_c is highly associated with the soil index properties PI and S200. This is not surprising since S200 and PI are the two most important variables in determining soil classification, and therefore X_c.

TABLE 4.6. Variance Inflation Factors: First Order Model, All Predictor Variables Included

Predictor Variable	Variance Inflation Factor (VIF)
R_{vm} (R-value modified)	5.50
σ_d (Deviator stress, psi)	1.00
σ_3 (Confining stress, psi)	1.00
w% (% water content)	32.07
γ_d (Dry density pcf)	15.14
S% (% Saturation)	13.89
PI (Plasticity Index)	6.64
S200 (% passing 200 screen)	5.11
X_c = 1 for clayey soil (A-7-6, A-6) otherwise X_c = 0	7.15

TABLE 4.7. Variance Inflation Factors: w% And γ_d Removed

Predictor Variable	Variance Inflation Factor
..... R_{vm} (Modified R-value)	5.33
..... σ_d (Deviator stress, psi)	1.00
..... σ_3 (Confining stress, psi)	1.00
S% (Degree of saturation, %)	4.24
PI (Plasticity Index)	2.92
S200 (Percent passing 200 screen)	3.87
X_c = 1 for clayey soils (A-7-6 and A-6) otherwise X_c = 0	6.62

TABLE 4.8. Correlation Matrix

No.	Variable	Y M_R	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_C
Y	M_R (Resilient Modulus)	1.000									
X_1	R_{VM} (R-value modified)	0.539	1.000								
X_2	σ_d (Deviator stress)	-0.121	0.023	1.000							
X_3	σ_3 (Confining stress)	0.105	-0.004	0.017	1.000						
X_4	w% (Water content %)	0.032	-0.577	0.009	-0.003	1.000					
X_5	Υ_d (Dry density)	-0.504	0.026	-0.004	0.005	-0.767	1.000				
X_6	S% (Degree of saturation)	-0.636	-0.861	-0.013	0.002	0.529	0.091	1.000			
X_7	PI (Plasticity Index)	0.302	-0.355	-0.013	-0.001	0.810	-0.793	0.134	1.000		
X_6	S200 (% passing 200 screen)	0.430	-0.130	0.008	0.007	0.654	-0.716	0.026	0.663	1.000	
X_C	(X_C = 1 for clayey soils, (X_C = 0 otherwise)	0.214	-0.414	-0.1019	-0.008	0.732	-0.645	0.207	0.812	0.827	1.000

It is desirable to select predictor variables that have a high degree of association with the response variable (resilient modulus) and, at the same time, less intercorrelation with other predictor variables. Two subset models were selected for further analysis based on: 1) a review of the bivariate correlations given in Table 4.8 and the analysis of variance given in Table 4.4, and 2) the primary objectives of this research. These objectives were to evaluate the relationship between R-value and resilient modulus and to develop an equation to predict resilient modulus as a function of easily measured soil index properties and specimen characteristics. The relationships that were evaluated are:

$$M_R = f(S\%, \sigma_d, \sigma_3, PI, S200) \tag{4.4}$$

$$M_R = f(R_{vm}, \sigma_d, \sigma_3, X_c) \tag{4.5}$$

Investigation of Subset Models

To investigate the two subset models, a stepwise regression analysis was performed to find the comparative significance of the individual variables. Several stepwise selection techniques were employed, including Maxr and Forward.

Maxr is a stepwise regression method that begins by finding the one variable model producing the highest correlation of determination (r^2). Then another variable, the one that yields the greatest increase in r^2, is added. The process is continued with comparisons at each step to check if removing any variable and replacing it with another variable increases r^2.

Stepwise regression using the forward-selection technique first incorporates the variables with the largest F statistic. Then it adds variables one at a time based on the F statistic until the p-values for the F statistic are greater than the level specified, in this case $\alpha = 0.05$. Results of the forward selection analysis, which yielded similar results to the Maxr analysis, are given in Tables 4.9 and 4.10.

TABLE 4.9. Stepwise Regression Analysis: Forward Selection Technique Subset Model 4.1 (See equation 4.1 for initial variables)

Step No.	Variable	B-Value	Std. Error	Variable F Statistic	p-value	Coeff. of Determination (r^2)
1.	Intercept	35566.73				
	S%	-337.22	26.29	164.52	0.0001	0.405
2.	Intercept	26221.71				
	S%	-345.73	21.34	262.44	0.0001	0.610
	S200	144.41	12.87	126.72	0.0001	
3.	Intercept	28545.64				
	S%	-347.27	20.82	278.15	0.0001	0.630
	S200	145.88	12.56	134.81	0.0001	
	σ_d	-301.33	82.73	13.27	0.0003	
4.	Intercept	30912.33				
	S%	-359.01	20.54	305.47	0.0001	0.650
	S200	107.36	16.08	44.57	0.0001	
	σ_d	-317.43	80.74	13.66	0.0001	
	PI	86.45	23.39	13.66	0.0003	
5.	Intercept	30278.38				
	S%	-359.22	20.20	315.96	0.0001	0.663
	S200	107.48	15.82	46.15	0.0001	
	σ_d	-325.05	79.48	16.73	0.0001	
	PI	86.36	23.01	14.09	0.0002	
	σ_3	236.82	79.28	8.92	0.0031	

TABLE 4.10. Stepwise Regression Analysis: Forward Selection Technique Subset Model 4.2 (See equation 4.2 for initial variables)

Step No.	Variable	B-Value	Std. Error	Variable F Statistic	p-value	Coeff. of Determination (r^2)
1.	Intercept	2524.53				
	R_{vm}	171.71	17.46	96.64	0.0001	0.290
2.	Intercept	-2762.88				
	R_{vm}	242.10	15.98	229.31	0.0001	0.514
	X_C	5471.90	526.06	108.19	0.0001	
3.	Intercept	-494.99				
	R_{vm}	242.83	15.67	240.24	0.0001	0.535
	X_C	5485.43	515.46	113.25	0.0001	
	σ_d	-307.93	93.80	10.78	0.0012	
4.	Intercept	-1187.86				
	R_{vm}	243.15	15.47	247.07	0.0001	0.549
	X_C	5506.10	508.10	117.02	0.0001	
	σ_d	-315.92	92.66	11.63	0.0008	
	σ_3	246.81	93.01	7.04	0.0085	

All of the variables initially considered in both subset models were found to be significant at $\alpha = .05$. The resulting regression equations are:

$$M_R = -1190 + 243*(R_{vm}) - 315*(\sigma_d) + 246*(\sigma_3) + 5510*(X_c) \qquad (4.6)$$

$r^2 = .549$

$$M_R = 30280 - 359*(S\%) - 325*(\sigma_d) + 237*(\sigma_3) + 86*(PI) + 107*(S200) \qquad (4.7)$$

$r^2 =.663.$

Figures 4.1 and 4.2 are plots of the residuals versus predicted values for equations 4.6 and 4.7, respectively. Both figures show that the error terms increase as Y (resilient modulus) increases. This increase is especially pronounced for equation 4.6, suggesting that a transformation of Y may reduce the increase in the error terms. For example, instead of regressing Y on the predictor variables, regress the natural logarithm of Y on the predictor variables. Figure 4.3 is a plot of the residuals versus predicted values for the natural log (Y) of equation 4.6. The randomness of the plot is substantially decreased.

Interaction and Nonlinear Terms. Stepwise regression was used to investigate the significance of interaction terms and transformations of the predictor variables for subset model 4.4. Additional test results, and test results beyond the range of the data generated in this study, would be useful for confirming nonlinear trends and interaction terms which were apparent in the analysis but not well defined. The regression parameters associated with PI*S200, $S200^2$, and PI^2 were found to be significant at $\alpha = .05$, and are recommended as topics for future research where the experimental design is set up specifically to evaluate these terms.

Final Models

From the analysis of variance, correlation matrix, variance inflation factors, and stepwise regression, the following two equations are proposed for estimating M_R:

$$LN(M_R) = 7.16 + 0.0389*(R_{vm}) - 0.049*(\sigma_d) + 0.040*(\sigma_3) + 1.01*(X_c) \qquad (4.8)$$

$r^2 = .593$

$$M_R = 30280 - 359*(S\%) - 325*(\sigma_d) + 236*(\sigma_3) + 86*(PI) + 107*(S200) \qquad (4.9)$$

$r^2 = .663.$

Tables 4.11 and 4.12 summarize the regression statistics for equations 4.8 and 4.9.

Evaluation of the Prediction Equations. Figures 4.4 and 4.5 are plots of the predicted values, based on equations 4.8 and 4.9, versus the observed values. The appearance of the plots does not indicate any significant trends that would suggest any further model development.

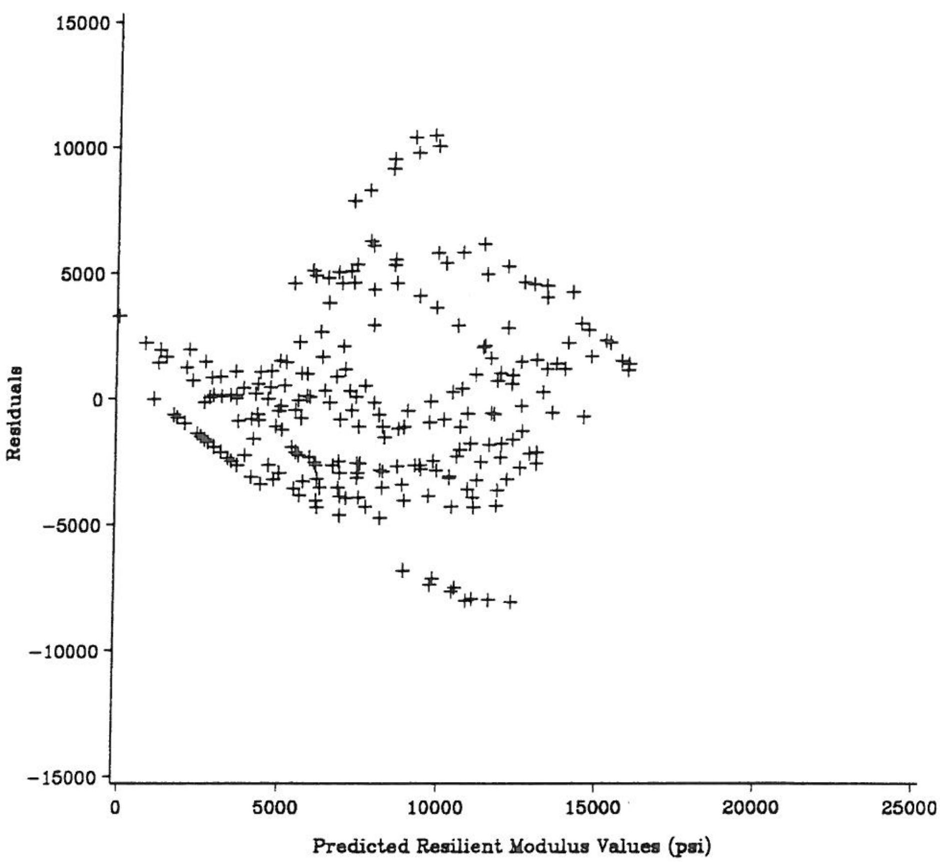

Figure 4.1. Residuals Versus Predicted Values for Equation 4.6.

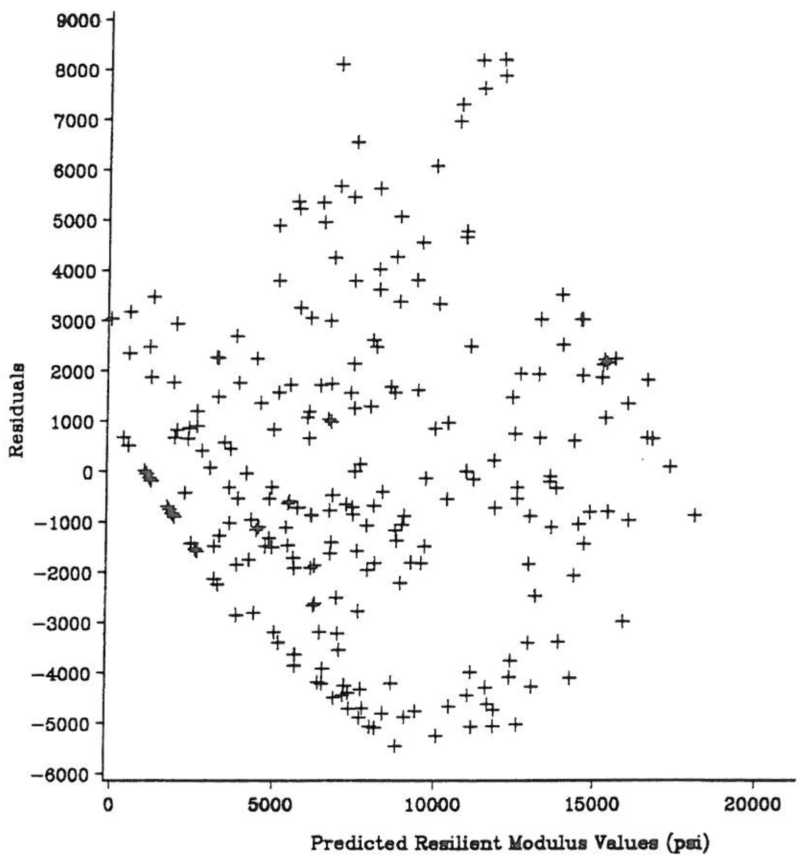

Figure 4.2. Residuals Versus Predicted Values for Equation 4.7.

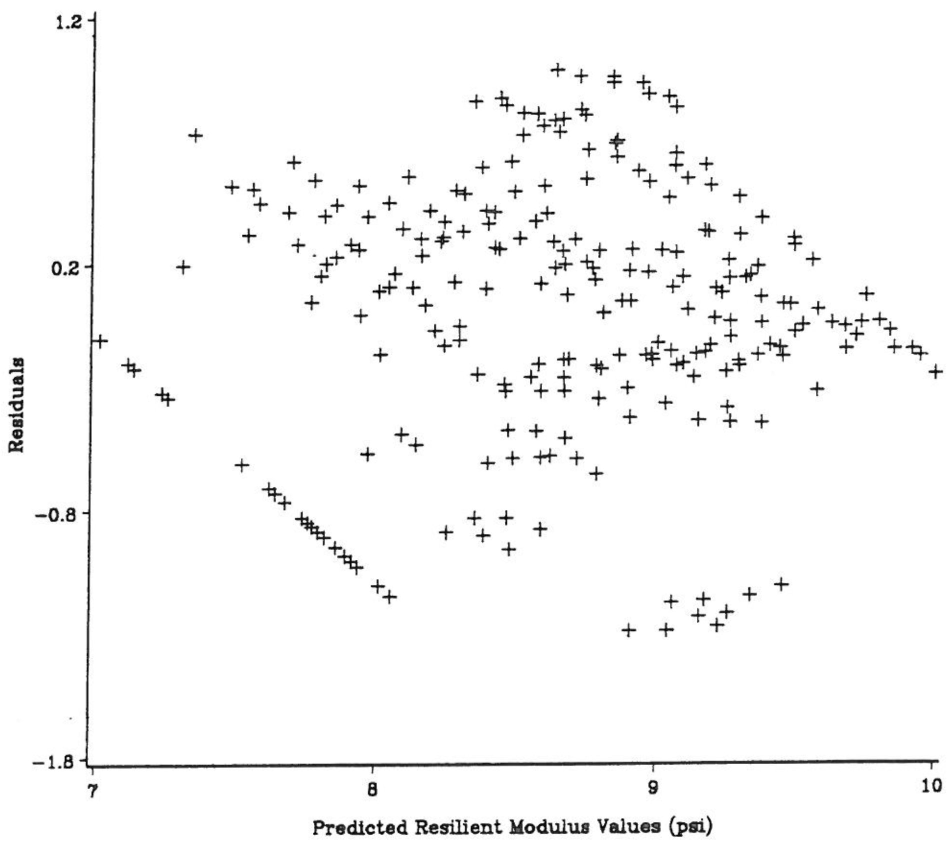

Figure 4.3. Residuals Versus Predicted Values for the Natural Log of M_R, Equation 4.6.

TABLE 4.11. Statistical Summary of Resilient Modulus, Equation 4.8

No.	Variable	Parameter Estimate	Std. Error	T for H_0: Parameter = 0	p-value
1.	Intercept	7.15	0.152	47.14	0.0001
2.	R_{vm}	0.039	0.002	16.65	0.0001
3.	X_C	1.013	0.077	13.17	0.0001
4.	σ_d	-0.049	0.014	-3.54	0.0005
5.	σ_3	0.040	0.014	2.84	0.0049

Analysis of Variance

Multiple Coefficient of Determination (r^2)	0.593
Adjusted Coefficient of Determination (r^2)	0.586
Standard Error of Estimate	0.531
F Statistic	84.99

TABLE 4.12. Statistical Summary of Resilient Modulus, Equation 4.9

No.	Variable	Parameter Estimate	Std. Error	T for H_0: Parameter = 0	p-value
1.	Intercept	30278.38	2036.69	14.87	0.0001
2.	S%	-359.22	20.21	-17.78	0.0001
3.	S200	107.48	15.82	6.79	0.0001
4.	PI	86.36	23.01	3.75	0.0002
5.	σ_d	-325.05	79.48	-4.09	0.0001
6.	σ_3	236.82	79.28	2.99	0.0031

Analysis of Variance

Multiple Coefficient of Determination (r^2)	0.662
Adjusted Coefficient of Determination (r^2)	0.655
Standard Error of Estimate	3034
F Statistic	93.6
p-value	0.0001

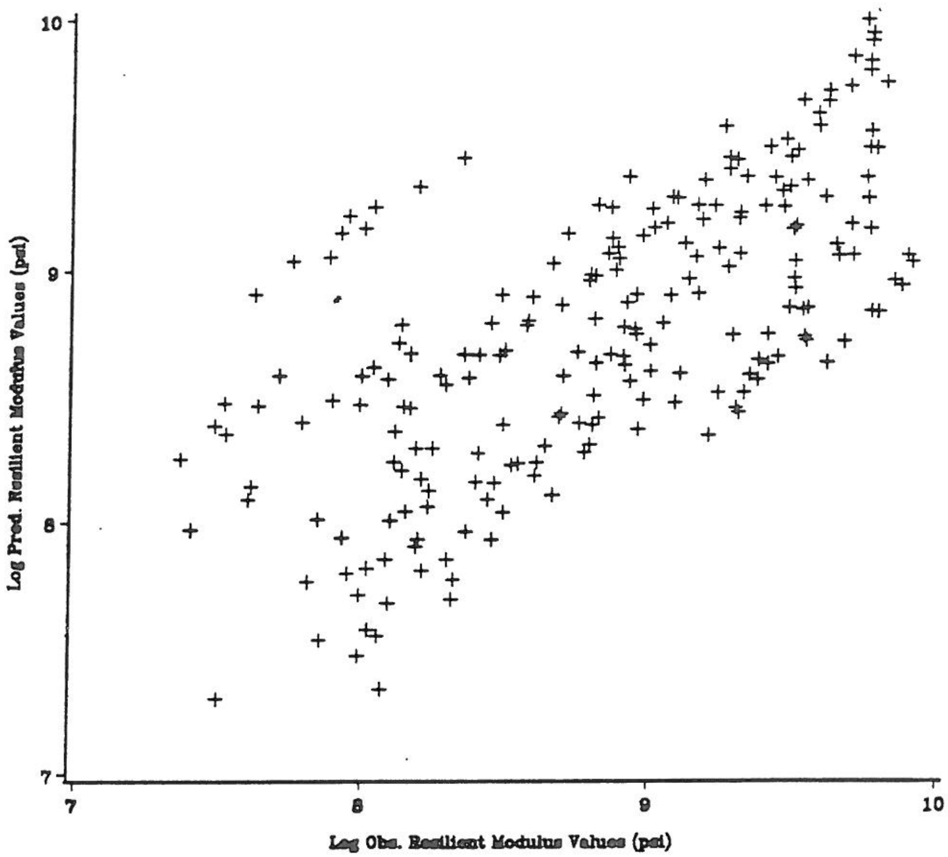

Figure 4.4. Predicted Values Versus Observed Values for Equation 4.8.

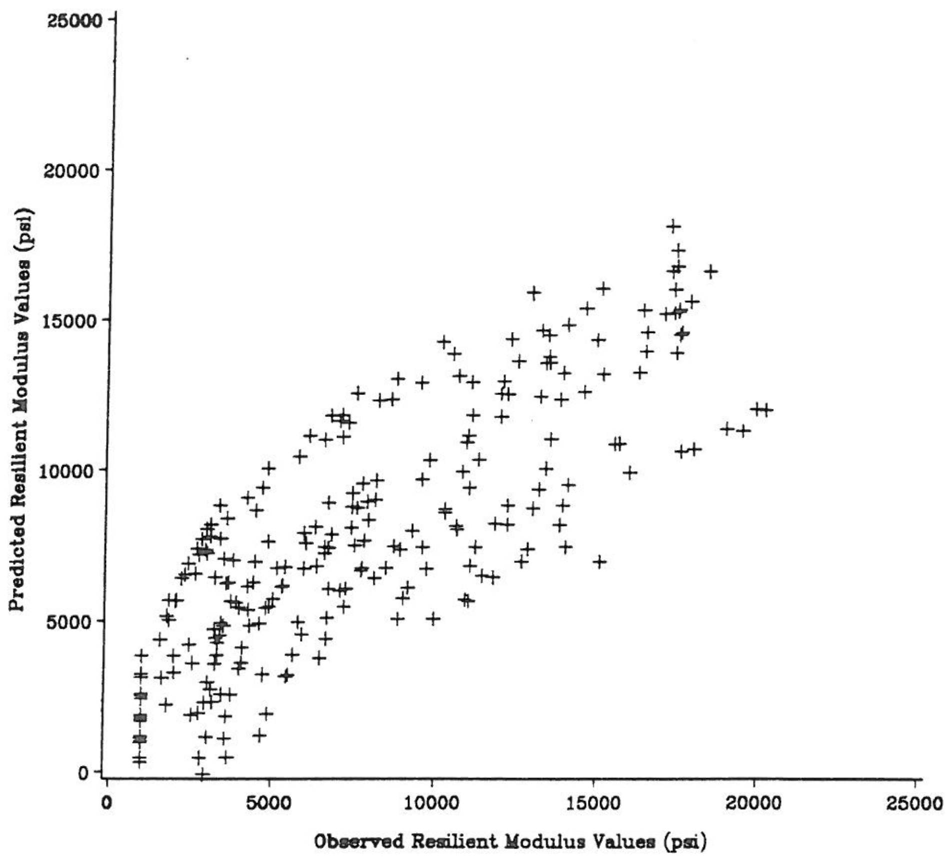

Figure 4.5. Predicted Values Versus Observed Values for Equation 4.9.

COMPARISON TO EXISTING PREDICTION MODELS

Several equations are presented in the literature for predicting resilient modulus as a function of R-value, degree of saturation, and/or other soil properties. These equations were reviewed in Chapter 2. Figure 4.6 compares three reported equations that predict M_R based on R-value to equation 4.8 of this study. Two special cases of equation 4.8 are plotted in order to match as closely as possible the conditions of the other studies. The first is for silty soils ($X_c = 0$) at a deviator stress of 4 psi and a confining stress of 2 psi. This curve yields values of M_R that are close to those predicted by the Idaho study, also conducted on silts. The curve of this study is more nonlinear and yields higher values of M_R as R-value increases. The second case is for clayey soils ($X_c = 1$) at a deviator stress of 4 psi and a confining stress of 2 psi. Comparing this curve to the Asphalt Institute study, equation 4.8 predicts lower values of M_R and shows a higher degree of nonlinearity. The Colorado study includes four different soil classification groups and predicts values of M_R that are intermediate between silts and clays.

Thompson and Robnett (1979) reported the following equation for fine-grained Illinois soils (see Chapter II):

$$M_R = 45200 - 428*(S\%) \qquad (2.17)$$

The relationship between M_R and S% for this study, at a deviator stress of $\sigma_d = 6$ psi and confining stress of $\sigma_3 = 0$, is given by:

$$M_R = 35100 - 335*(S\%) \qquad (4.10)$$

$r^2 = .428$, $S_e = 3,950$

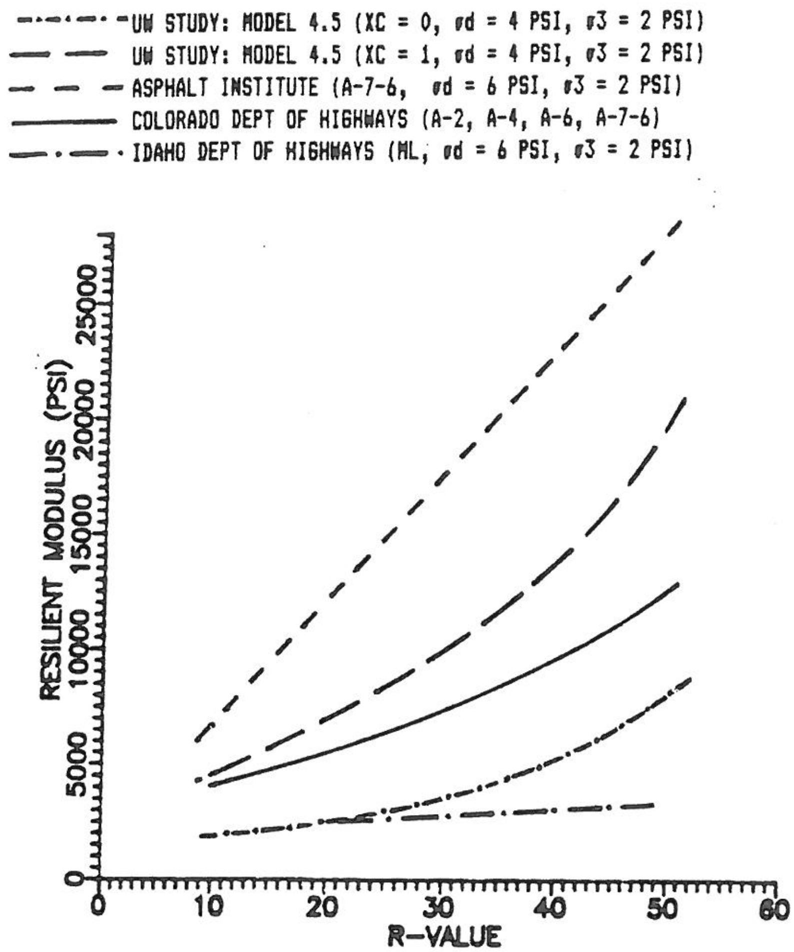

Figure 4.6. Resilient Modulus Versus R-Value. Comparison of Equation 4.8 to Other Reported Relationships.

In the Thompson-Robnett study, the mean plasticity index was PI = 16.7. The mean percent passing the No. 200 was S200 = 89.7 percent. Figure 4.7 compares the Thompson-Robnett model (equation 2.17) to equation 4.10 at three different percentages passing the No. 200 sieve (S200 = 50, 70, and 90 percent) and a constant plasticity index (PI = 16.7). For 90 percent passing the No. 200 sieve, the relationship between M_R and degree of saturation is almost identical to the Thompson and Robnett equation. The results of this study indicate that the resilient modulus decreases with decreasing percentage of fines, for a constant plasticity index and for a given degree of saturation.

SUMMARY

A statistical analysis of laboratory test results, including resilient modulus, R-value, soil index properties, and sample characteristics, yielded two equations which predict the resilient modulus of subgrade soils (equations 4.8 and 4.9). A comparison of these prediction equations to published equations for resilient modulus is presented and shows general agreement with previous work, although some differences, including a higher degree of nonlinearity, are apparent. The advantage of the prediction equations developed by this study is that they are based upon tests conducted on soils obtained from actual highway pavement construction projects in Wyoming.

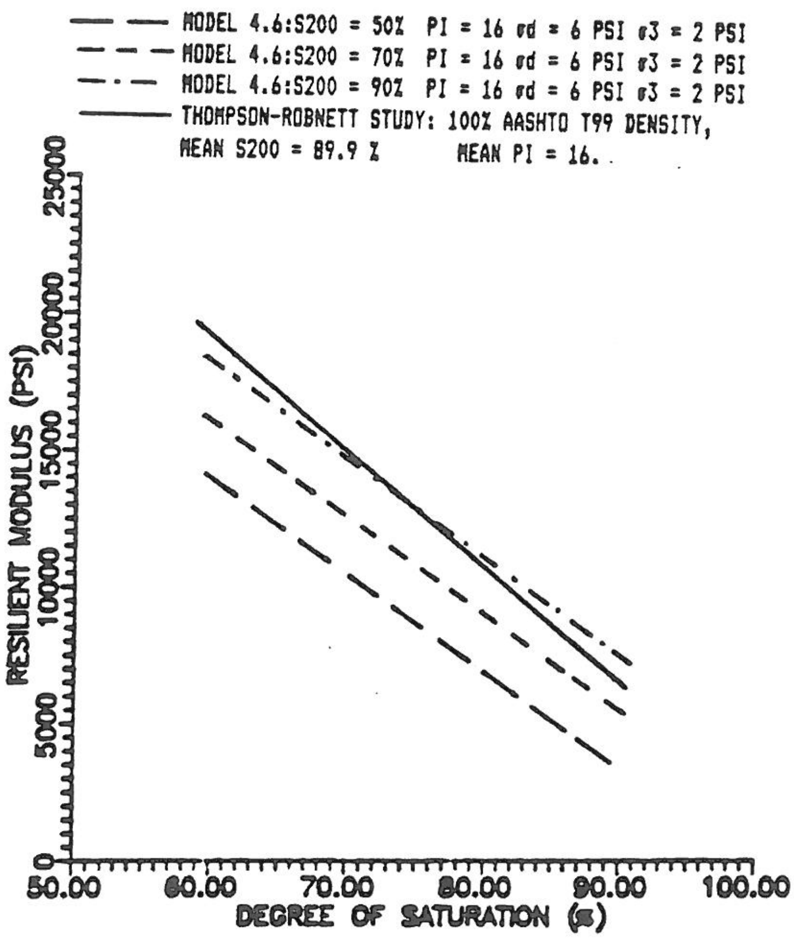

Figure 4.7. Resilient Modulus Versus Degree of Saturation: Comparison of Equation 4.9 to Thompson & Robnett (1979).

CHAPTER 5

SUMMARY AND CONCLUSIONS

SUMMARY

A study was conducted to: a) investigate the relationship between resilient modulus and R-value for representative Wyoming soils and b) develop a regression model to predict the resilient modulus of fine-grained soils (clays and silts) based on easily measured soil index properties and specimen characteristics. A laboratory study was conducted in which thirteen fine-grained Wyoming soils were evaluated by three types of tests:

1) The repeated load triaxial test (AASHTO T274),
2) The Standard R-value test (AASHTO T190), and
3) A modification of the standard R-value test.

Soil index properties and specimen characteristics such as degree of saturation also were measured and recorded.

Nine predictor variables were selected initially for a tentative first-order regression model for predicting resilient modulus. Standard R-value test results were not selected since a majority of the results were reported as -5. Initial predictor variables selected were:

S% = degree of saturation (%);

R_{vm} = modified R-value (0-100 scale);

σ_d = deviator stress (psi);

σ_3 = confining stress (psi);

PI = plasticity index;

S200 = percent passing No. 200 (% by weight);

$w\%$ = moisture content (% by weight);

γ_d = dry density (pcf);

X_c = 1 for Clay soils (A-7-6, A-6), and $X_c = 0$ otherwise.

Statistical analysis of the model, incorporating all of the above predictor variables, indicated severe multicollinearity among several of the predictor variables. Moisture content and dry density were dropped from the model to reduce the degree of multicollinearity. Two subset models were selected for further investigation based on the degree of collinearity between the predictor variables. Stepwise regression analysis was used to evaluate the significance of the predictor variables for each subset model.

Final Models

Two multiple linear regression models are proposed for estimating resilient modulus. The models explain 66 and 59 percent of the variation, respectively, from the observed values of resilient modulus (29 specimens at various levels of deviator and confining stress). The two equations, in recommended order are:

$$M_R = 34280 - 359*(S\%) - 325*(\sigma_d) + 236*(\sigma_3) + 86(PI) + 107*(S200) \qquad (4.9)$$

$r^2 = 0.663$, $S_e = 3{,}034$ psi

$$LN(M_R) = 7.157 + 0.039*(R_{vm}) - 0.049*(\sigma_d) + 0.040*(\sigma_3) + 1.013*(X_c) \qquad (4.8)$$

$r^2 = 0.593$, $S_e = .531$

where: $LN(M_R)$ = natural logarithm resilient modulus (psi)

In Chapter 4, the above two equations were compared to several other well-known models for predicting resilient modulus. The results compare favorably, however, since the soils used in this study were obtained from Wyoming Highway Department pavement construction sites, the results are believed to be more representative of Wyoming conditions.

Linear Relationships. Resilient modulus was found to be strongly negatively associated with degree of saturation (r = -0.636) and to a lesser extent with deviator stress (r = -0.121). Resilient modulus was found to be positively associated with modified R-value (r = 0.539), percent passing the No. 200 (r = 0.430), plasticity index (r = 0.302), and confining stress (r = 0.105).

Influence of Soil Type. The mean resilient response of the clayey soil samples (AASHTO Soil Classifications A-6 and A-7-6) was M_R = 8,770 psi. The mean resilient response of the silty soil samples (AASHTO Soil Classification A-4) was M_R = 6,530 psi. The difference was significant at $\alpha = 0.001$.

CONCLUSIONS

The principal result of this research is the development of improved equations for predicting the resilient modulus of soils in Wyoming. These equations yield resilient modulus as a function of soil type, the state of stress in the pavement subgrade, the degree of soil saturation, R-value, and other soil index properties. Although further verification is needed before implementing these results into pavement design procedures, this study represents the first step in providing highway engineers in Wyoming with reliable information on the resilient modulus of representative subgrade soils. This information can be used to make informed decisions on how to incorporate the recommendations of the 1986 AASHTO Guide for Design of Pavement Structures into pavement design methodology.

RECOMMENDATIONS FOR FURTHER RESEARCH

1) Additional resilient modulus and R-value laboratory test results, especially for silty soils, would be useful for verification and to test the general applicability of the regression models developed in this study to other fine-grained soils in Wyoming. This additional work is presently being undertaken by the first author at the Wyoming Highway Department.

2) Pavement design engineers invariably are faced with estimating the final in-service subgrade soil conditions, especially with regard to the degree of saturation. This is true regardless of whether resilient modulus for subgrade soils is determined by AASHTO procedure T274, by the prediction equations presented herein, or by other published models. A review and analysis of the literature on estimating in-service subgrade soil conditions, and perhaps the collection of additional data, would be useful to assess the level of confidence achievable in estimating in-service conditions.

3) The following two suggestions are offered for modifying the AASHTO T274 procedure with respect to fine-grained soils: 1) eliminate unrealistic stress states such as σ_d =10 psi and σ_3 = 0 psi, and 2) rather than require 200 load repetitions for every stress state, simply require that a stable hysteresis condition be achieved.

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Appendix A

Resilient Modulus Test Results

SAMPLE NO.	(PSI) σ_3	(PSI) σ_d	RESILIENT MODULUS (PSI)
CR(14+16) +1% e - Testing Error w = 20.4 % γ_d = 101.4 pcf	6	4	e
	3	4	e
	0	4	20200
	6	8	19900
	3	8	19500
	0	8	18000
	6	10	19000
	3	10	17600
	0	10	16000
CR(14+16) +2% w = 23.0 % γ_d = 99.7 pcf	6	4	15660
	3	4	e
	0	4	e
	6	8	14080
	3	8	13920
	0	8	12220
	6	10	12220
	3	10	11840
	0	10	11240
TM(8+9) -1% w = 10.4 % γ_d = 125.0 pcf	6	4	6360
	3	4	3590
	0	4	e
	6	8	5010
	3	8	4610
	0	8	3290
	6	10	5800
	3	10	e
	0	10	4000
TM(8+9) 300 w = 11.5 % γ_d = 123.3 pcf	6	4	4270
	3	4	3240
	0	4	2470
	6	8	4090
	3	8	3270
	0	8	3050
	6	10	4070
	3	10	3150
	0	10	2950
TM(8+9) +2% w = 13.7 % γ_d =119.3 pcf * - Excessive Plastic Strain > 4%	6	4	2580
	3	4	e
	0	4	1800
	6	8	3190
	3	8	*
	0	8	*
	6	10	*
	3	10	*
	0	10	*

SAMPLE NO.	(PSI) σ_3	(PSI) σ_d	RESILIENT MODULUS (PSI)
CR(6+31) -1% e - Testing Error w = 17.2 % γ_d = 105.2 pcf	6	4	e
	3	4	e
	0	4	e
	6	8	13900
	3	8	12000
	0	8	12000
	6	10	12200
	3	10	11100
	0	10	11000
CR(6+31) +2% w = 21.2 % γ_d = 103.1 pcf	6	4	10810
	3	4	7450
	0	4	10280
	6	8	7590
	3	8	6810
	0	8	6600
	6	10	5970
	3	10	e
	0	10	5130
TM(4+6) -1% w = 16.4 % γ_d = 110.9 pcf	6	4	13830
	3	4	14020
	0	4	15080
	6	8	12660
	3	8	11800
	0	8	11040
	6	10	11460
	3	10	10930
	0	10	9960
TM(4+6) +2 w = 18.8% γ_d = 108.0 pcf	6	4	8140
	3	4	9020
	0	4	8870
	6	8	6690
	3	8	6660
	0	8	6480
	6	10	5920
	3	10	5640
	0	10	5440
TM(7+16) -1% w = 17.5 % γ_d = 106.6 pcf	6	4	15530
	3	4	13390
	0	4	13180
	6	8	7750
	3	8	7430
	0	8	7420
	6	10	6710
	3	10	6320
	0	10	6010

SAMPLE NO.	(PSI) σ_3	(PSI) σ_d	RESILIENT MODULUS (PSI)
TM(7+16) +2% e - Testing Error w = 18.9 % γ_d = 104.0 pcf	6	4	4830
	3	4	4660
	0	4	4470
	6	8	3380
	3	8	2980
	0	8	2960
	6	10	3100
	3	10	2690
	0	10	2420
TM(14+15) +2% w = 15.0 % γ_d = 116.6 pcf	6	4	4210
	3	4	3610
	0	4	2840
	6	8	3100
	3	8	2760
	0	8	2330
	6	10	3000
	3	10	2650
	0	10	2040
TM(31+32) -1% w = 14.5 % γ_d = 108.1 pcf	6	4	12280
	3	4	12510
	0	4	12070
	6	8	8770
	3	8	8230
	0	8	7300
	6	10	8600
	3	10	7030
	0	10	6570
TM(31+32) +2 w = 17.4 % γ_d = 110.1 pcf	6	4	7510
	3	4	7750
	0	4	6740
	6	8	4240
	3	8	3980
	0	8	3440
	6	10	3900
	3	10	3520
	0	10	3340
TM(17+18) -1% w = 11.2 % γ_d = 118.5 pcf	6	4	13000
	3	4	10660
	0	4	8920
	6	8	12850
	3	8	9720
	0	8	7260
	6	10	11080
	3	10	9160
	0	10	7210

SAMPLE NO.	(PSI) σ_3	(PSI) σ_d	RESILIENT MODULUS (PSI)
TM(17+18) +2% e - Testing Error w = 14.4 % γ_d = 115.5 pcf * EXCESSIVE PLATIC STRAIN >4%	6	4	*
	3	4	*
	0	4	*
	6	8	*
	3	8	*
	0	8	*
	6	10	*
	3	10	*
	0	10	*
TM(17+18) 300 w = 13.0 % γ_d = 118.5 pcf	6	4	e
	3	4	3330
	0	4	2020
	6	8	4720
	3	8	3470
	0	8	2790
	6	10	e
	3	10	3620
	0	10	3040
ALF CORE w = 17.1 % γ_d = 114.3 pcf	6	4	8150
	3	4	7950
	0	4	7810
	6	8	4860
	3	8	4460
	0	8	4420
	6	10	3800
	3	10	3660
	0	10	3740
TM(34+38) -1% w = 11.2 % γ_d = 113.2 pcf	6	4	13820
	3	4	e
	0	4	10940
	6	8	13530
	3	8	9790
	0	8	8180
	6	10	11320
	3	10	9560
	0	10	7910
TM(34+38) +2% w = 14.2 % γ_d = 114.2 pcf *EXCESSIVE PLASTIC STRAIN > 4%	6	4	3420
	3	4	2000
	0	4	1640
	6	8	*
	3	8	*
	0	8	*
	6	10	*
	3	10	*
	0	10	*

SAMPLE NO.	(PSI)		RESILIENT MODULUS (PSI)
	σ_3	σ_d	
TM(34+38) 300 e - Testing Error w = 13.5 % γ_d = 117.9 pcf	6	4	5480
	3	4	3760
	0	4	2560
	6	8	4870
	3	8	3590
	0	8	2840
	6	10	4680
	3	10	3670
	0	10	2960
ALF PIT +2% w = 19.4 % γ_d = 108.8 pcf	6	4	3410
	3	4	3520
	0	4	3260
	6	8	2230
	3	8	2070
	0	8	1850
	6	10	1840
	3	10	1780
	0	10	1580
ALF PIT -1% w = 16.5 % γ_d = 107.3 pcf	6	4	10490
	3	4	10670
	0	4	13190
	6	8	7540
	3	8	7100
	0	8	7120
	6	10	6770
	3	10	6090
	0	10	5790
TM(14+15) -1% w = 11.7 % γ_d = 113.4 pcf	6	4	17230
	3	4	e
	0	4	18420
	6	8	17420
	3	8	15070
	0	8	17500
	6	10	e
	3	10	14590
	0	10	13220
MV(6) -1% w = 15.3 % γ_d = 106.1 pcf	6	4	17340
	3	4	17060
	0	4	17530
	6	8	17580
	3	8	17420
	0	8	15130
	6	10	16460
	3	10	16260
	0	10	14550

SAMPLE NO.	(PSI) σ_3	(PSI) σ_d	RESILIENT MODULUS (PSI)
MV(6) +2 e - Testing Error w = 18.1 % γ_d = 110.2 pcf	6	4	6610
	3	4	6720
	0	4	5970
	6	8	5390
	3	8	5280
	0	8	4800
	6	10	5310
	3	10	4900
	0	10	4310
MV(4) -1% w = 12.6 % γ_d = 108.5 pcf	6	4	17850
	3	4	14020
	0	4	10150
	6	8	14940
	3	8	13360
	0	8	9510
	6	10	13480
	3	10	11080
	0	10	e
MV(4) +2% w = 15.2 % γ_d = 110.4 pcf	6	4	11040
	3	4	10280
	0	4	9280
	6	8	10630
	3	8	8730
	0	8	7720
	6	10	9590
	3	10	8500
	0	10	7090
MV(4) w = 12.0 % γ_d = 107.5	6	4	17400
	3	4	17270
	0	4	12920
	6	8	17350
	3	8	17480
	0	8	13440
	6	10	16380
	3	10	16490
	0	10	13450

Appendix B

Scatter Plots

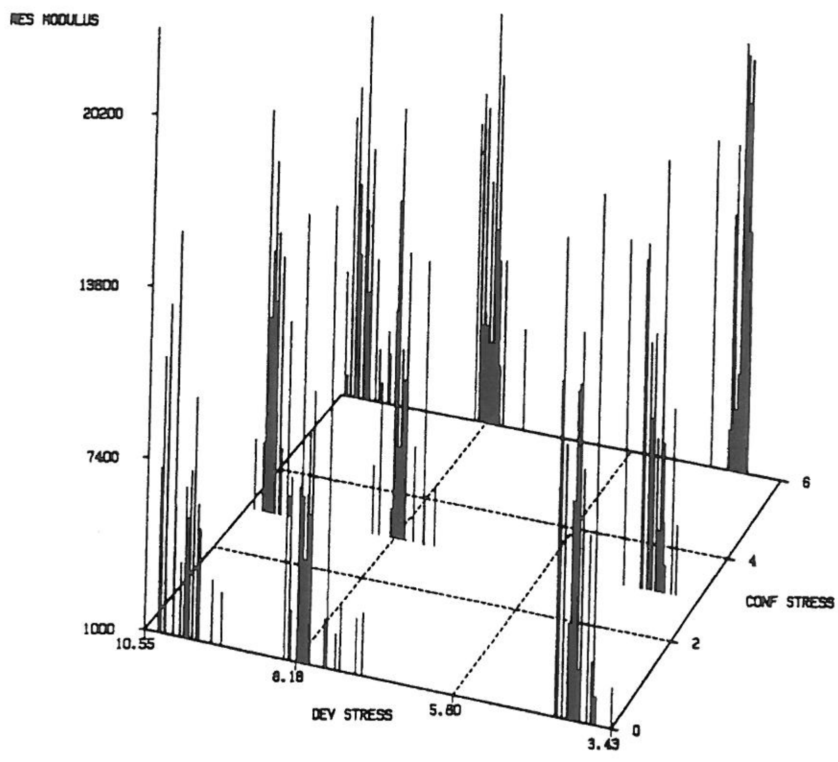

Figure B-1. 3-Dimensional Scatter Plot of Resilient Modulus (psi) and Deviator Stress (psi) Versus Confining Stress (psi).

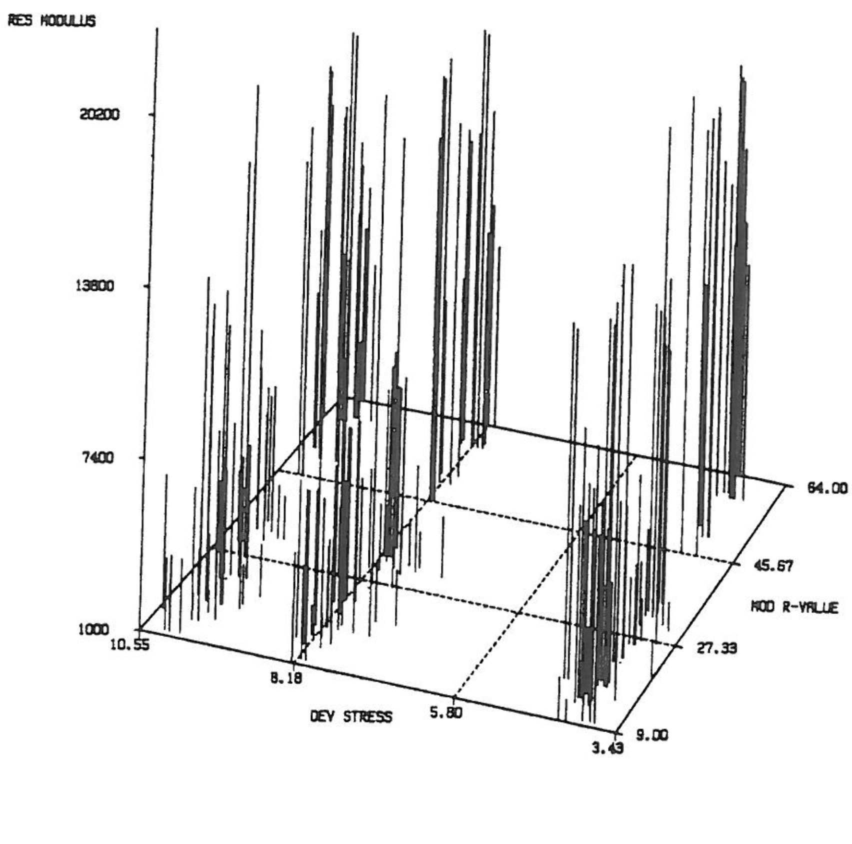

Figure B-2. 3-Dimensional Scatter Plot of Resilient Modulus (psi) and Deviator Stress (psi) Versus Modified R-Value.

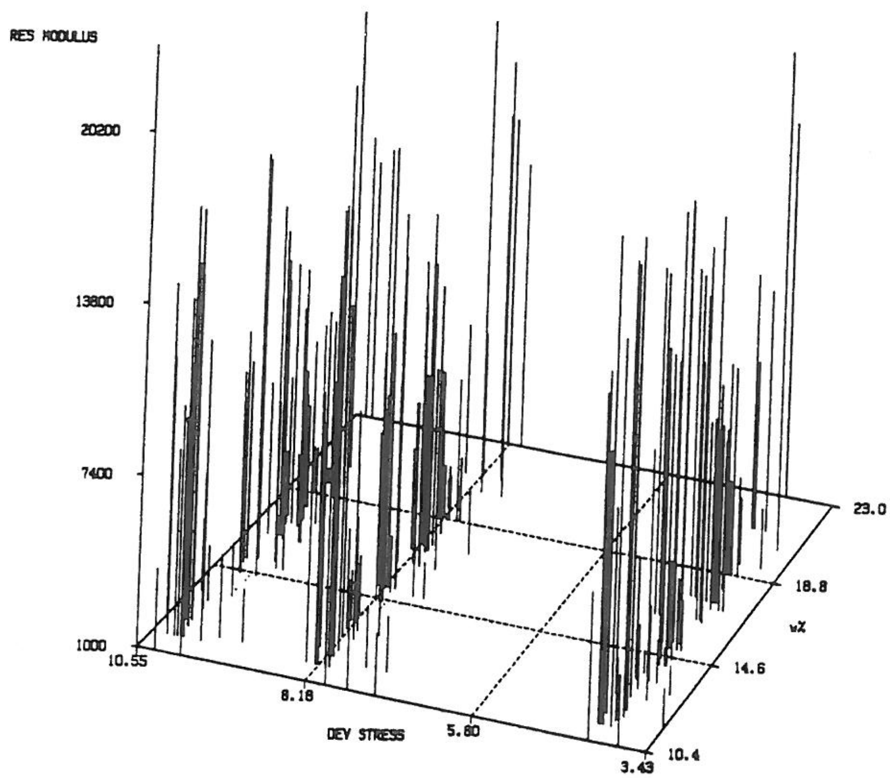

Figure B-3. 3-Dimensional Scatter Plot of Resilient Modulus (psi) and Deviator Stress (psi) Versus Moisture Content (%).

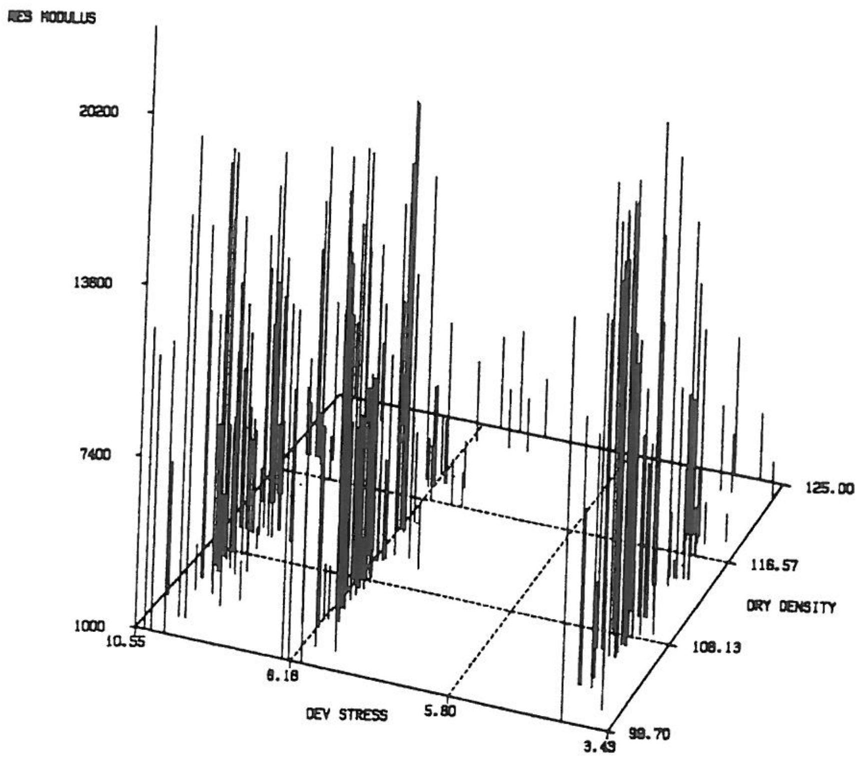

Figure B-4. 3-Dimensional Scatter Plot of Resilient Modulus (psi) and Deviator Stress (psi) Versus Dry Density (pcf).

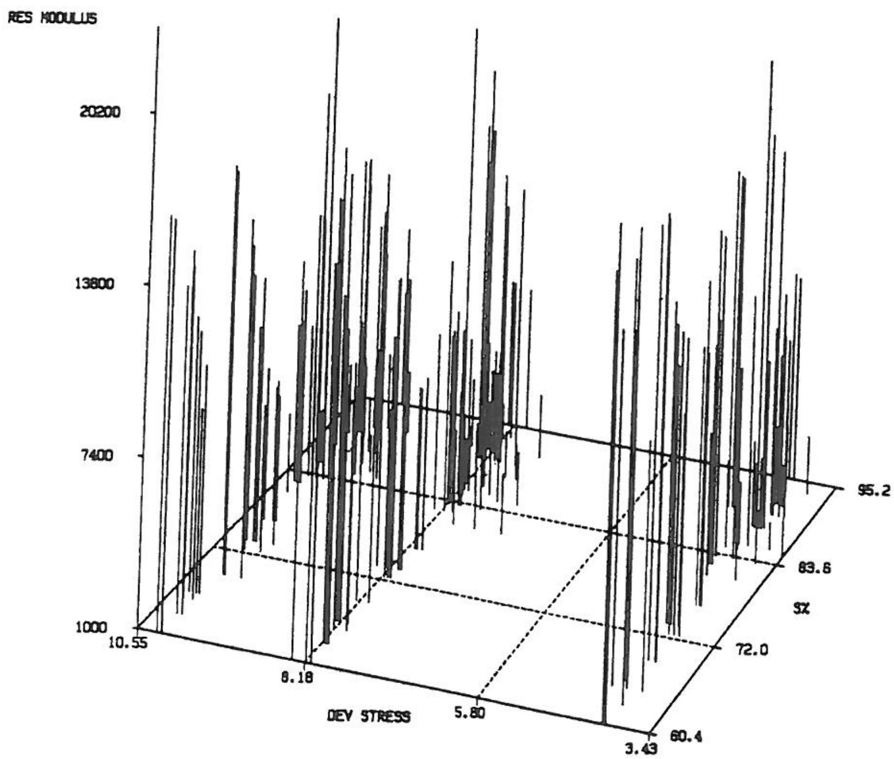

Figure B-5. 3-Dimensional Scatter Plot of Resilient Modulus (psi) and Deviator Stress (psi) Versus Degree Of Saturation (%).

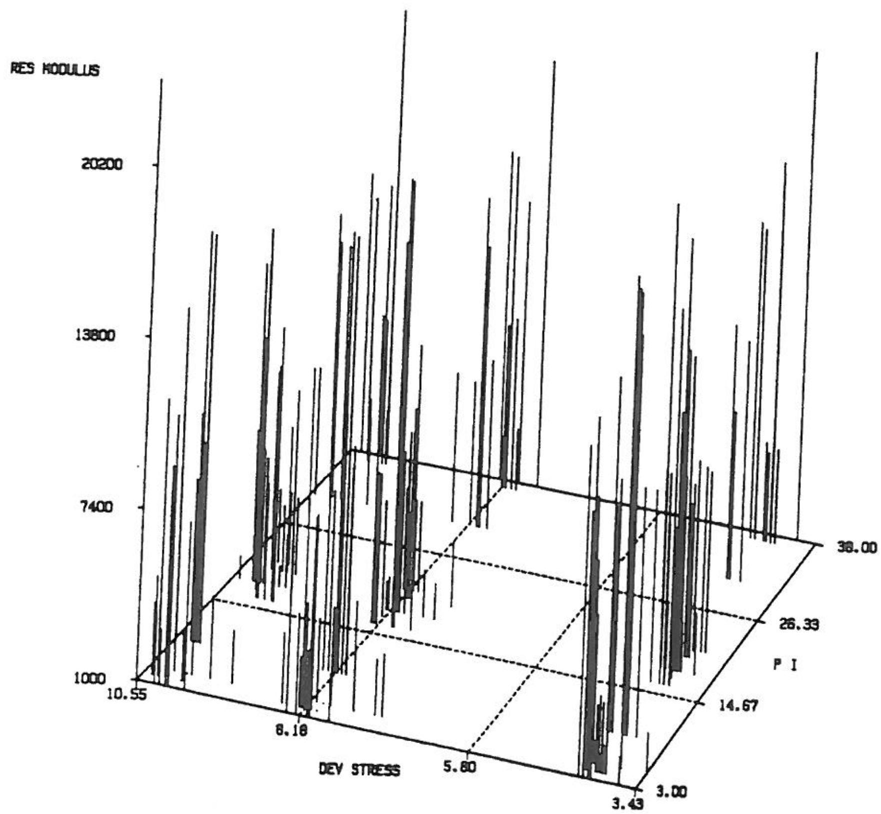

Figure B-6. 3-Dimensional Scatter Plot of Resilient Modulus (psi) and Deviator Stress (psi) Versus Plasticity Index.

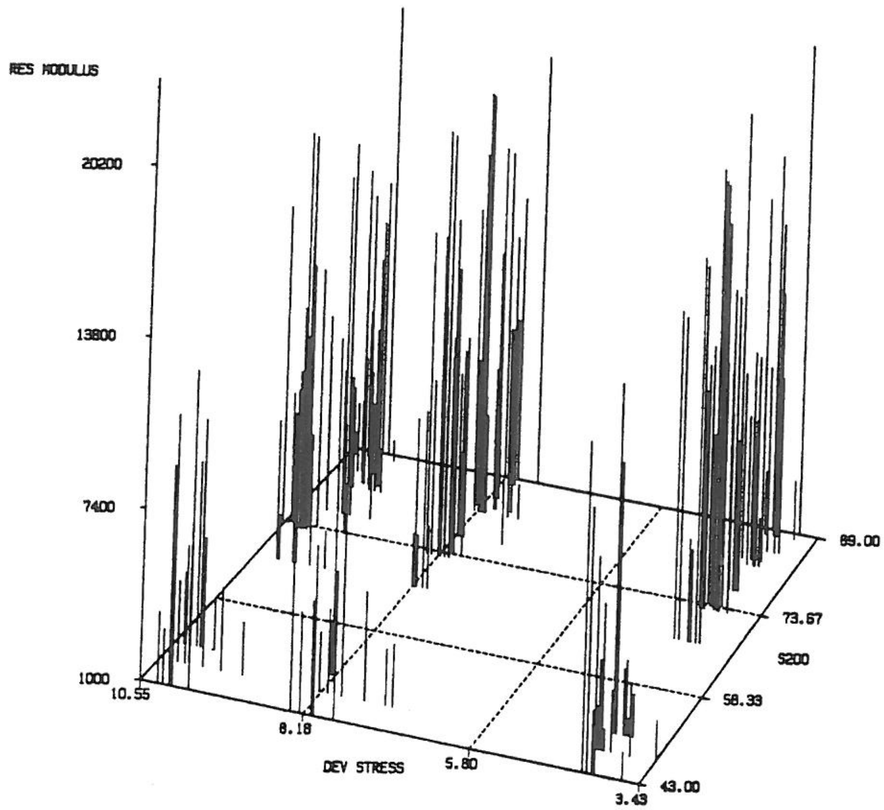

Figure B-7. 3-Dimensional Scatter Plot of Resilient Modulus (psi) and Deviator Stress (psi) Versus Percent Passing the No. 200 Sieve.

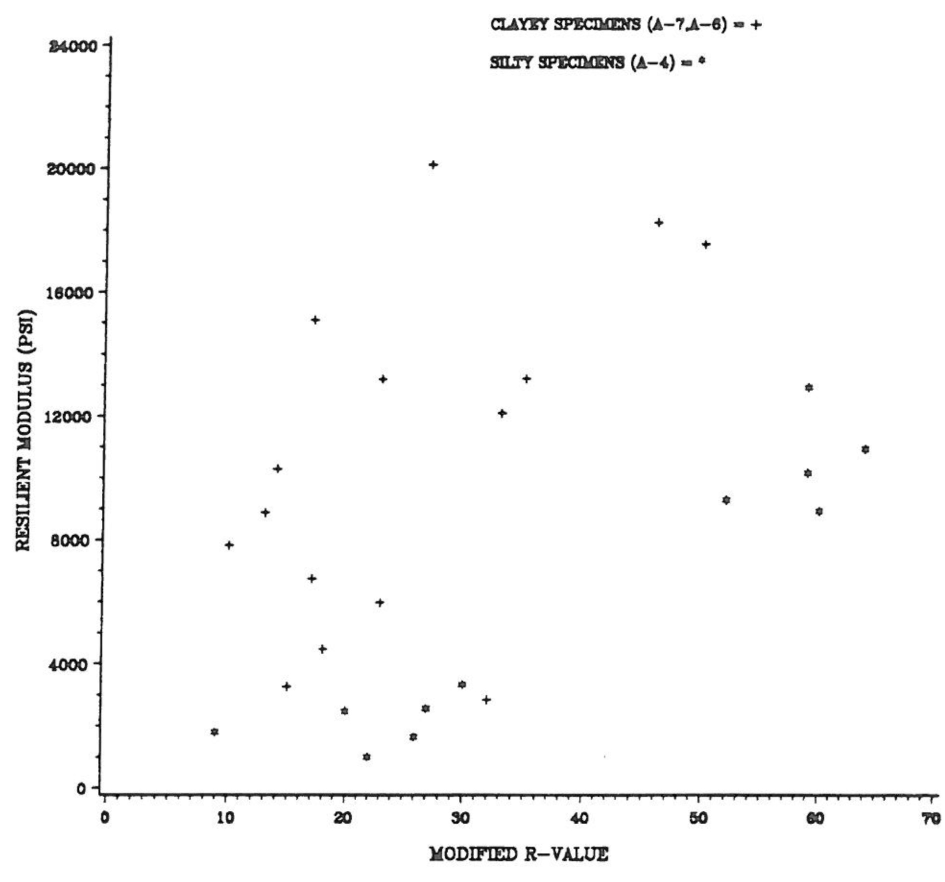

Figure B-8. Scatter Plot of Resilient Modulus Versus Modified R-Value at Constant Deviator and Confining Stress (σ_d = 4 psi, σ_3 = 0 psi).

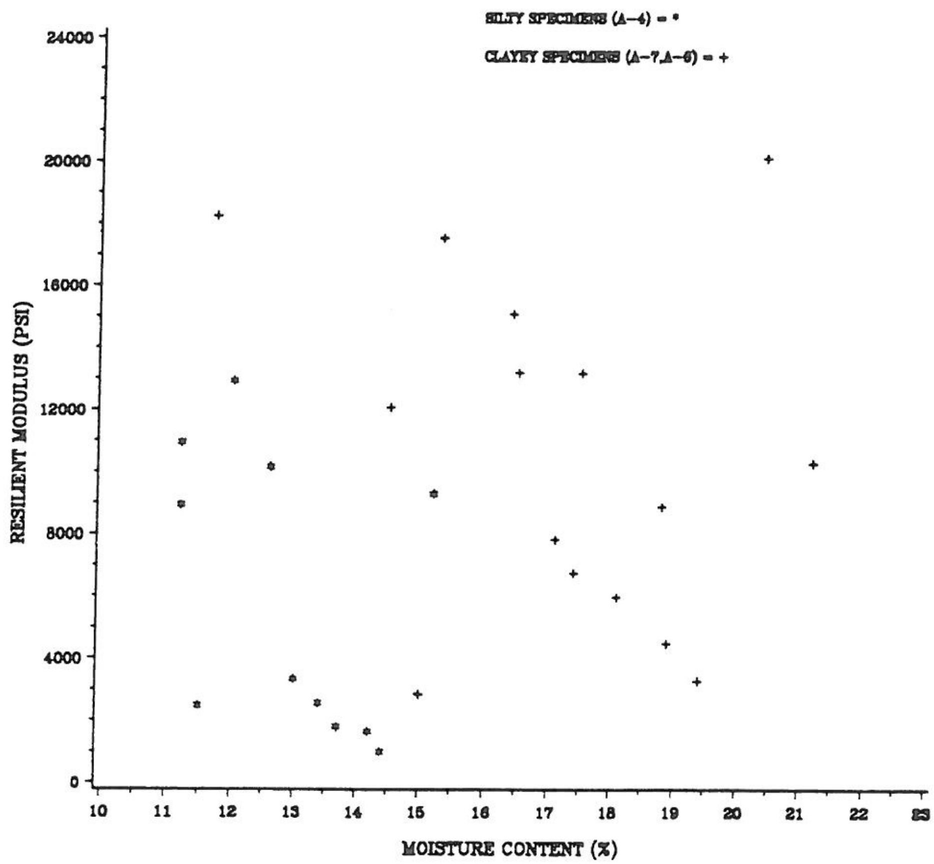

Figure B-9. Scatter Plot of Resilient Modulus Versus Moisture Content at Constant Deviator and Confining Stress (σ_d = 4 psi, σ_3 = 0 psi).

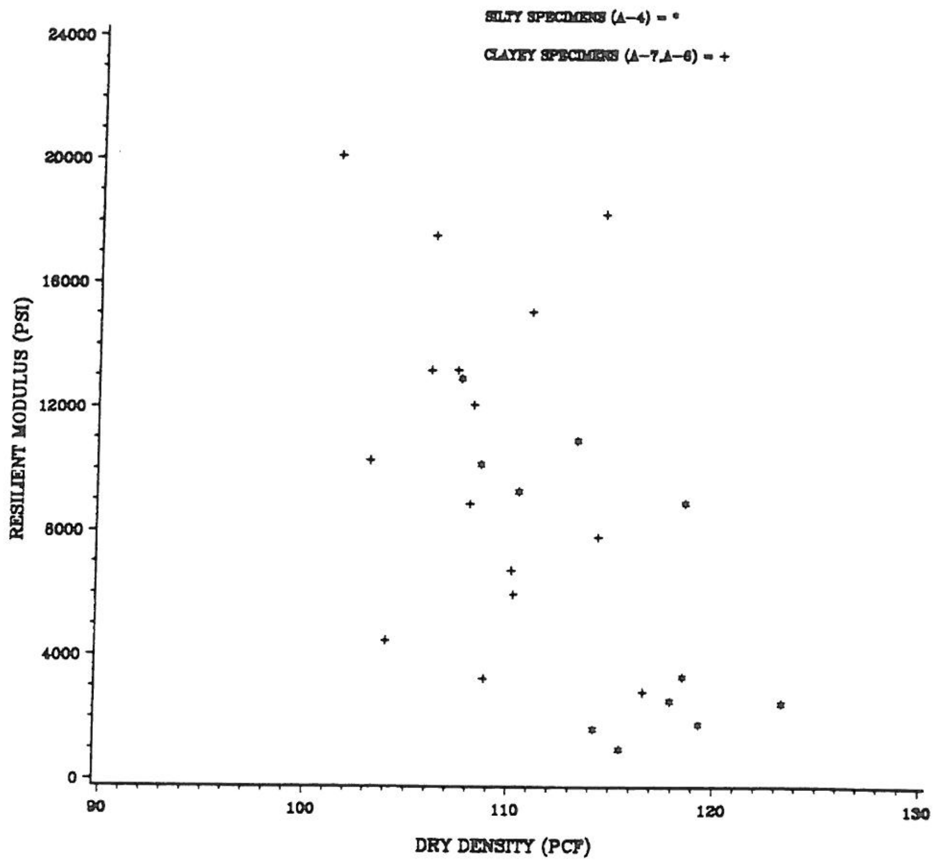

Figure B-10. Scatter Plot of Resilient Modulus Versus Dry Density at Constant Deviator and Confining Stress (σ_d = 4 psi, σ_3 = 0 psi).

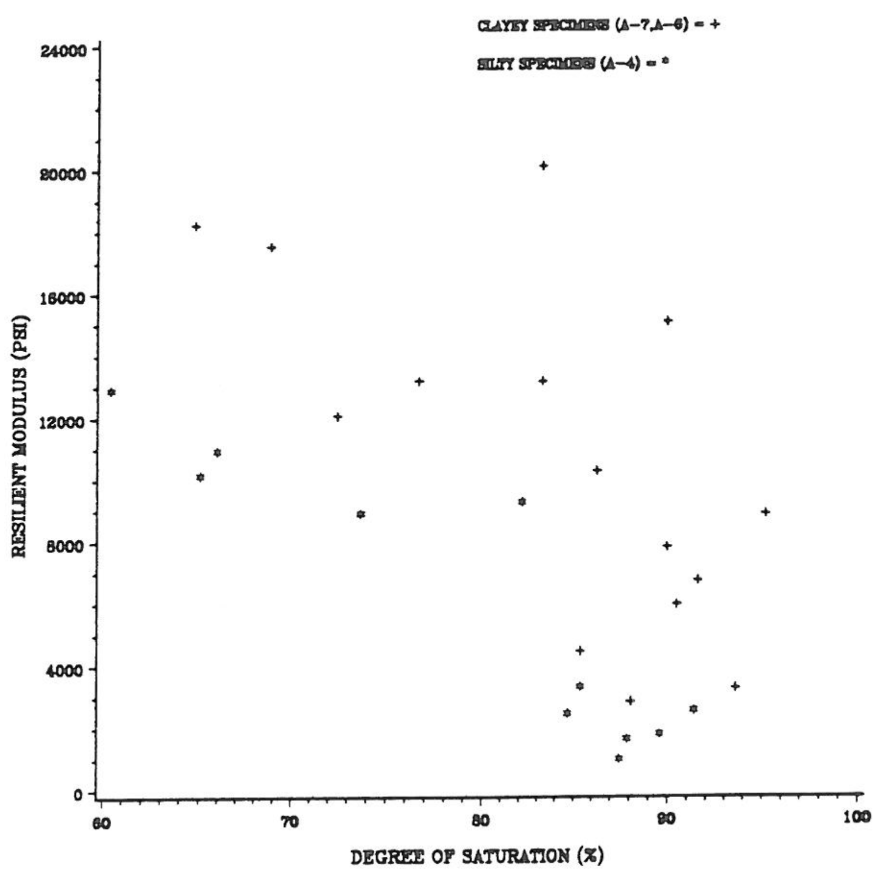

Figure B-11. Scatter Plot of Resilient Modulus Versus Degree of Saturation at Constant Deviator and Confining Stress (σ_d = 4 psi, σ_3 = 0 psi).

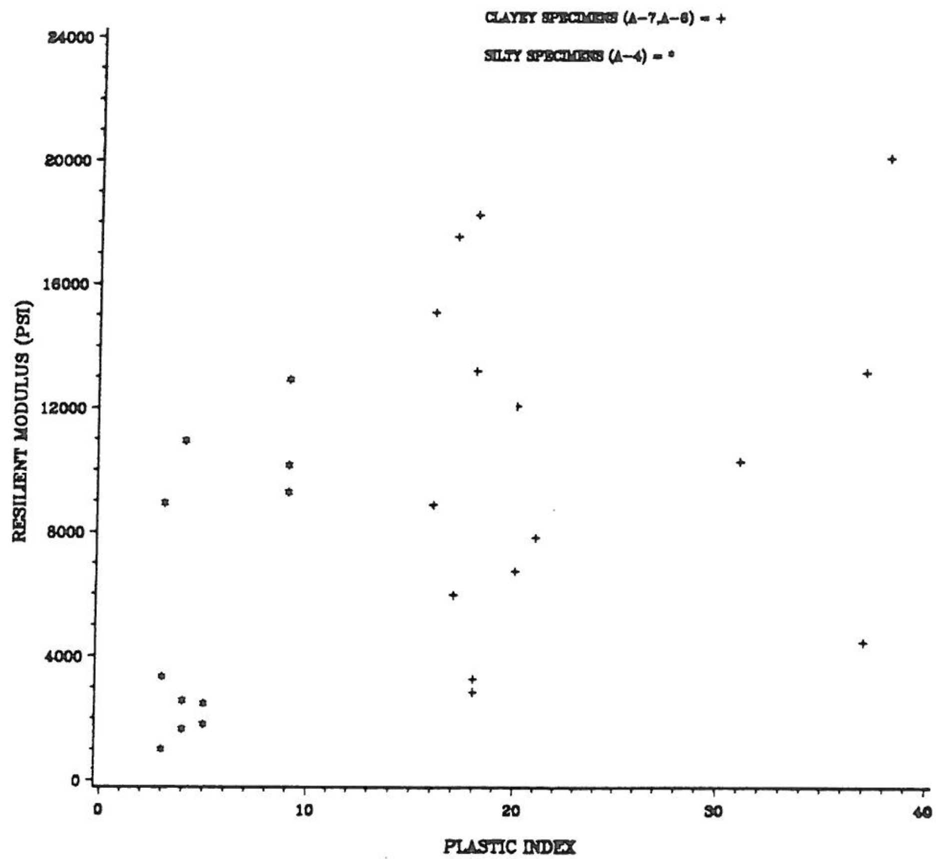

Figure B-12. Scatter Plot of Resilient Modulus Versus Plasticity Index at Constant Deviator and Confining Stress (σ_d = 4 psi, σ_3 = 0 psi).

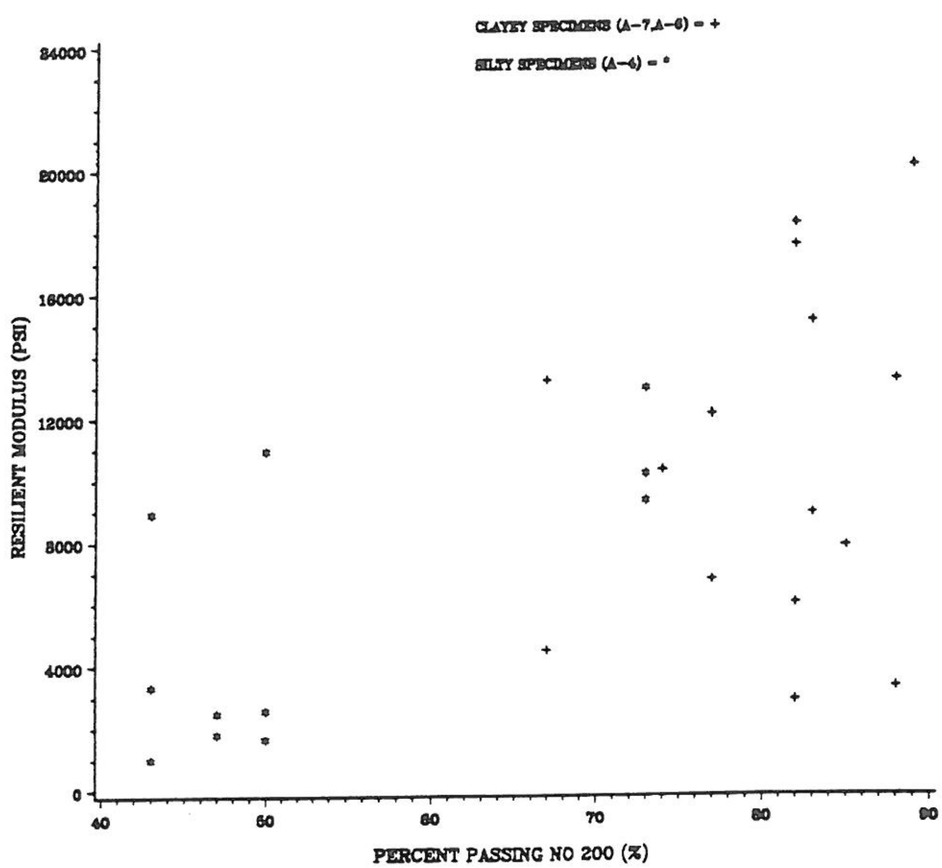

Figure B-13. Scatter Plot of Resilient Modulus Versus Percent Passing the No. 200 at Constant Deviator and Confining Stress (σ_d = 4 psi, σ_3 = 0 psi).