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A Bicycle Network
Analysis Tool for Planning
Applications in Small
Communities



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**A Bicycle Network Analysis Tool for Planning Applications
in Small Communities**

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1. INTRODUCTION

1.1 Background

Non-motorized transportation modes such as bicycles constitute an important part of a community's transportation system; they are vital to the success of transit-oriented developments (TODs). However, bicycles were often ignored in transportation planning and in travel demand forecasting modeling. At best, they were treated as a byproduct in the planning process. In addition, many cities have begun to invest and promote cycling as a healthy, environmentally friendly, and economical alternative mode of travel to motorized vehicles (especially private motorized vehicles). However, the current practice in modeling bicycle trips in a network is inadequate. Only a few research efforts focus on network analysis for bicycle trips (e.g., Klobucar and Fricker, 2007; Broach et al., 2011; Mekuria et al., 2012). These methods provide an initial effort to develop a traffic assignment method for bicycle trips, but they are too simplistic (i.e., simply based on all-or-nothing (AON) assignment method using a single attractiveness measure (e.g., distance, safety, or a composite measure of safety multiplied by distance).

Compared with route choice behavior for drivers of private motorized vehicles, route choice behavior for cyclists is much more complex; there are many influential factors affecting cyclist route choice decisions. Many empirical studies on bicycle route choice analysis indicate that cyclists choose routes based on a number of criteria (e.g., distance, number of intersections, road grade, bike facility, safety, etc.). Due to a diverse set of influential factors in bicycle travel, many route planners provide a variety of bicycle routes based on different factors (e.g., least elevation gain route, shortest distance route, safest route, least accident route, bike friendly route, lowest pollution route, route with green space, etc.) to satisfy the requirements of different cyclists (see Table 1.1). Note that all these provided routes are based on a single objective (i.e., shortest path based on distance or safest route based on some measure of safety).

Table 1.1 Online bicycle trip planners

Route planner	Provided routes
Los Angeles Route Planner (http://opt.berkeley.edu)	<ul style="list-style-type: none"> • Avoiding elevation gain • Avoiding pollution • Preferring green space • Avoiding prior bicycle accidents
San Francisco Bicycle Trip Planner (http://amarpai.com/bikemap)	<ul style="list-style-type: none"> • Shortest path • Balanced route • Bike-friendly route • Restrictions on gradient
Sacramento Region Bicycle Trip Planner (http://www.sacregion511.org/bicycling/trips)	<ul style="list-style-type: none"> • Shortest path • Bike-friendly route
Vancouver Cycle Trip Planner (http://cyclevancouver.ubc.ca)	<ul style="list-style-type: none"> • Shortest path • Least traffic pollution • Least elevation gain • Vegetated path • Restrictions on gradient
Washington D.C. Bike Planner (http://bikeplanner.org)	<ul style="list-style-type: none"> • Shortest path • Least elevation gain • Bike-friendly route
New York City Bike Map (http://www.nyc.gov/html/dot/html/bicyclists/bike_maps.shtml)	<ul style="list-style-type: none"> • Shortest path • Safe route • Safer route

1.2 Objectives

The overall goal of this research is to develop network analysis tools for estimating bicycle trips in small communities with limited resources. Specifically, the objectives include the following:

1. Collect bicycle data (facility and field data) from different sources to construct a bicycle network in a geographical information system (GIS) framework.
2. Develop a bicycle O-D demand generation procedure for generating an initial bicycle O-D matrix.
3. Develop a bicycle traffic assignment procedure for estimating bicycle volumes on a transportation network.
4. Develop a bicycle O-D demand adjustment procedure for refining the initial bicycle O-D matrix to better match with the observed bicycle counts.
5. Conduct a case study.

1.3 Organization of the Report

The organization of this report is summarized as follows:

- Section 2 describes the data collection for conducting a bicycle network analysis.
- Section 3 presents an initial bicycle O-D demand generation procedure.
- Section 4 presents the two-stage bicycle traffic assignment procedure.
- Section 5 presents the bicycle O-D demand adjustment procedure.
- Section 6 summarizes the findings and recommendations for future research.

2. DATA COLLECTION

2.1 Bicycle Facility Data

The term “bicycle facility” refers to all facilities that may be affected by bicycle travel. This includes network infrastructure such as a bikeway and bicycle parking lots. A bikeway is a general term for a route, lane, or path for bicycle travel. A bike route is a street signed for bicycle use, but it is not exclusive to bicycles because the roadway will be shared with motor vehicle traffic. Bike lanes and bike paths are exclusive to bicycles, but they differ in terms of proximity to motorized vehicular traffic. Since bike lanes are a marked section of the roadway, cyclists are close to motorized vehicles. In contrast, since bike paths are completely physically separated from motorized vehicle traffic, cyclists are relatively farther away from motorized vehicles. Figure 2.1 shows the different types of bikeways.

Bicycle parking is an important factor in promoting bicycle travel. The space, location, and security of bicycle parking could encourage bicycle travel as well as provision of bicycle facilities. These facilities may include lockers and weatherproofing roofs over bicycle parking structures. Figure 2.2 shows the different type of bicycle parking lots.

Facility Type	Definition	Example
Bike route	A street signed for bicycle use. Bicycles will share the roadway with motor vehicle traffic.	
Bike lane	A portion of a roadway which has been designated by striping, signing, and pavement markings for the preferential or exclusive use of bicyclists	
Bike path	A bikeway physically separated from motorized vehicular traffic by an open space or barrier; it is either within the highway right-of-way or within an independent right-of-way.	

Definition source: http://ridethisbike.com/bicycle_trails/bicycle_trail-bikeway_definitions.htm

Figure 2.1 Different types of bikeways (Source: City of Chicago)



Figure 2.2 Different types of bicycle parking facilities

2.2 Other Data

There are numerous factors that deter bicycle travel. Based on bicycle level of service measures (BLOS) in the Highway Capacity Manual (HCM, 2010), the factors can be classified with respect to network infrastructure and motorized flow. Table 2.1 summarizes the related factors in terms of these two categories.

Table 2.1 Influential factors in bicycle travel

Network infrastructure	Motorized flow
<ul style="list-style-type: none"> • average effective width of outside through lane • total number of directional through lanes • pavement surface condition rating • crossing distance • number of unsignalized intersections 	<ul style="list-style-type: none"> • proportion of heavy motorized vehicles • effective speed factor • directional motorized vehicle volume

2.3 Bike Facilities in Cache County, Utah

Local communities proposed the Bike Route Vision Plan for Cache County. These routes are mostly located on existing roads distinguished by road signage (Cache County, 2012). Figure 2.3 shows the proposed “on-road” Bike Route Vision Plan for Cache County.

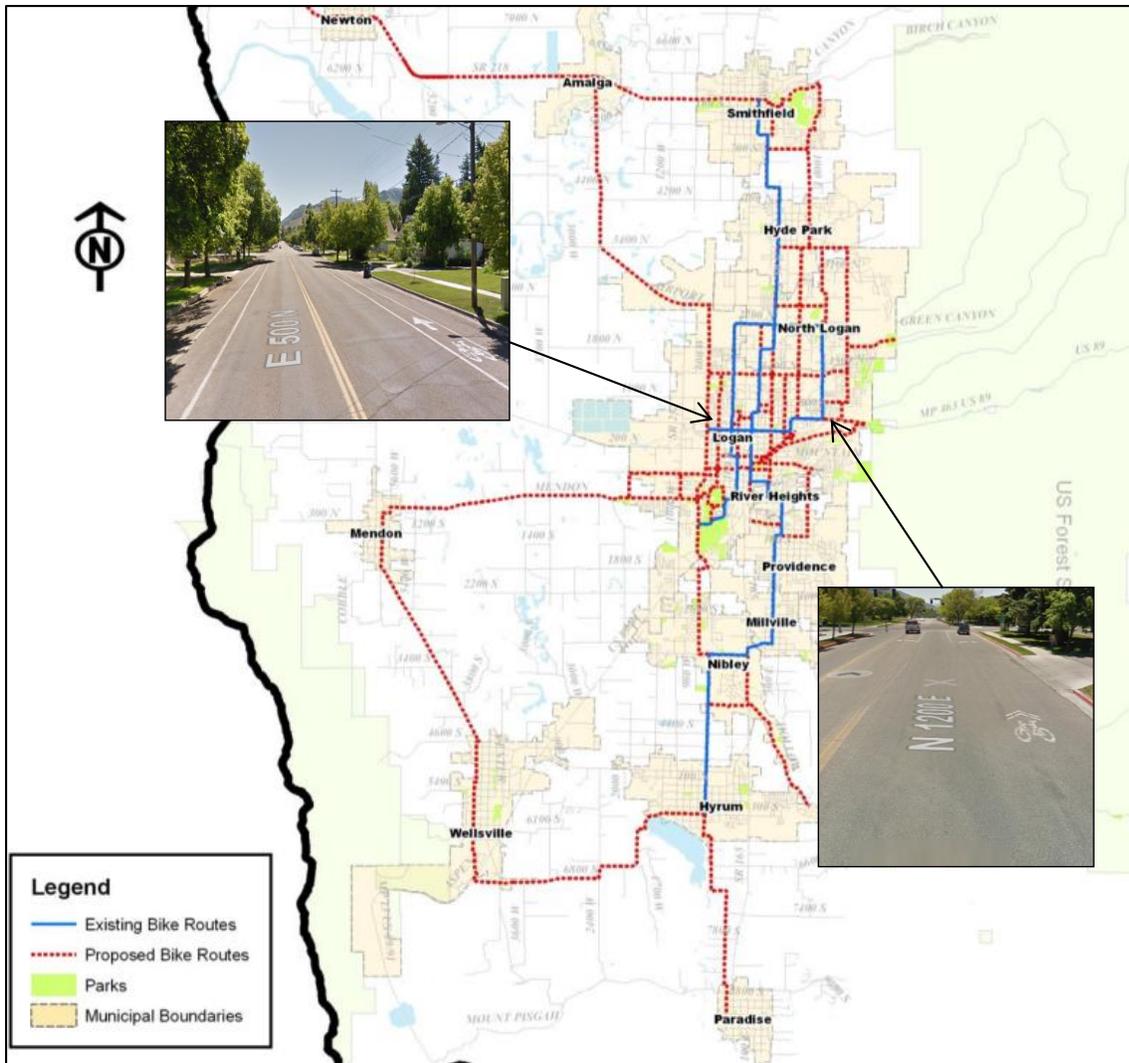


Figure 2.3 Cache County “on-road” Bike Route Vision Plan

2.4 Bicycle Data from Utah State University

Figure 2.4 shows an aerial overview map of Utah State University (USU) and marks major student residential areas, bicycle parking lots, and bikeways. For scale, the university spans about 500 acres (2.0 km²), in Logan, Utah, and is home to more than 14,000 students who live on or near campus. In terms of bicycle parking, most buildings on campus feature bicycle parking structures that hold a range of 10 to 100 individual parking facilities. The residential area data in Figure 2.4 are based on student dormitory data for both campus and off-campus housing, and the bikeway data (e.g., bike lane) are gathered from the 2035 Regional Transportation Plan (Cache County, 2012).

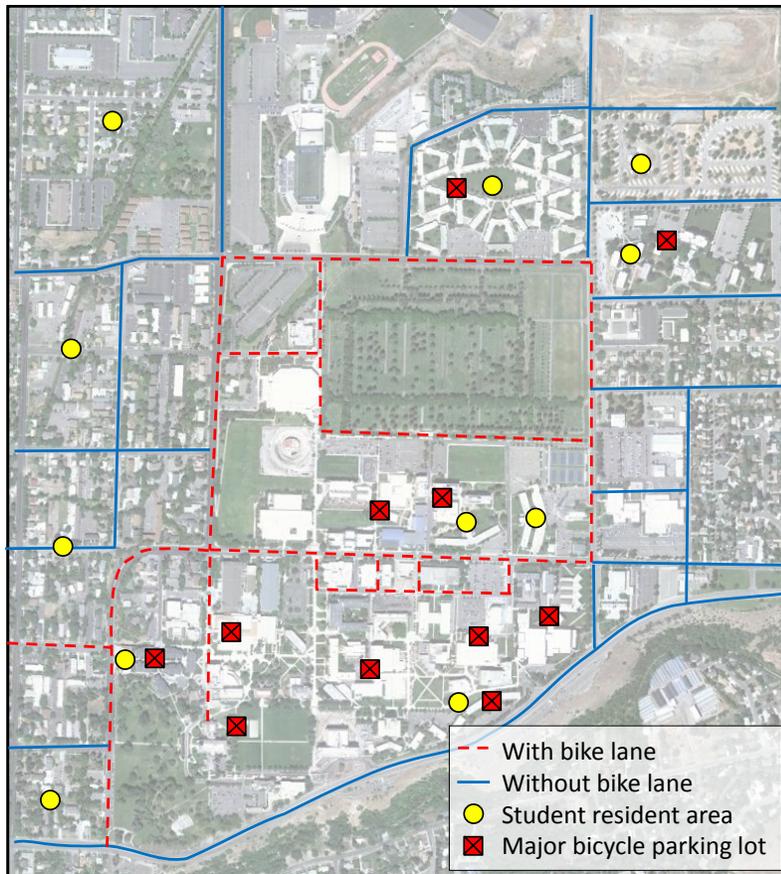
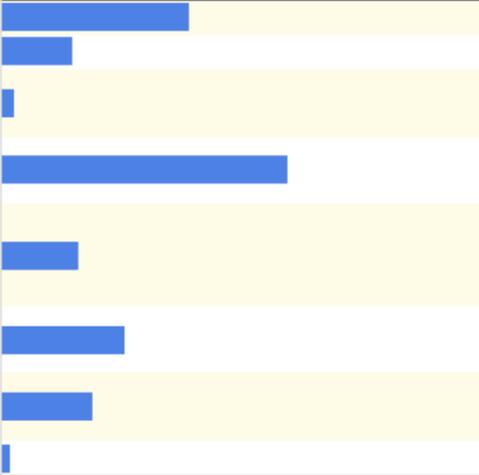


Figure 2.4 Map of Utah State University with bicycle facilities

2.5 2014 Transportation Survey in Utah State University

USU performed a transportation survey in 2014 to assess student needs for transportation and to improve student transportation in terms of shuttle buses and bicycle parking lots. A total of 4,469 students (about 26% of enrollment) participated in the survey. Detailed results are provided below in Table 2.2, Figure 2.5, and Figure 2.6. Table 2.2 describes USU student mode choice upon arrival to campus with a variety of mode choice selections: walking, biking, riding motorcycles or scooters, driving, and using public transit. Figure 2.5 and Figure 2.6 give a geographic overview of campus-bound bicycle trips. Note that the survey results are extracted based on the students' relation to bicycle transportation. Overall results are referred in the 2014 Transportation Survey Results (Utah State University, 2014).

Table 2.2 Mode choice with arriving on campus

#	Answer		Response
1	Walk		1,733
2	Bike		649
3	Motorcycle or Scooter		130
4	Car-single Occupancy		2,619
5	Car-multiple Occupancy (Carpool)		716
6	Campus Shuttle System		1,133
7	Cache Valley Transit		848
8	Other		78

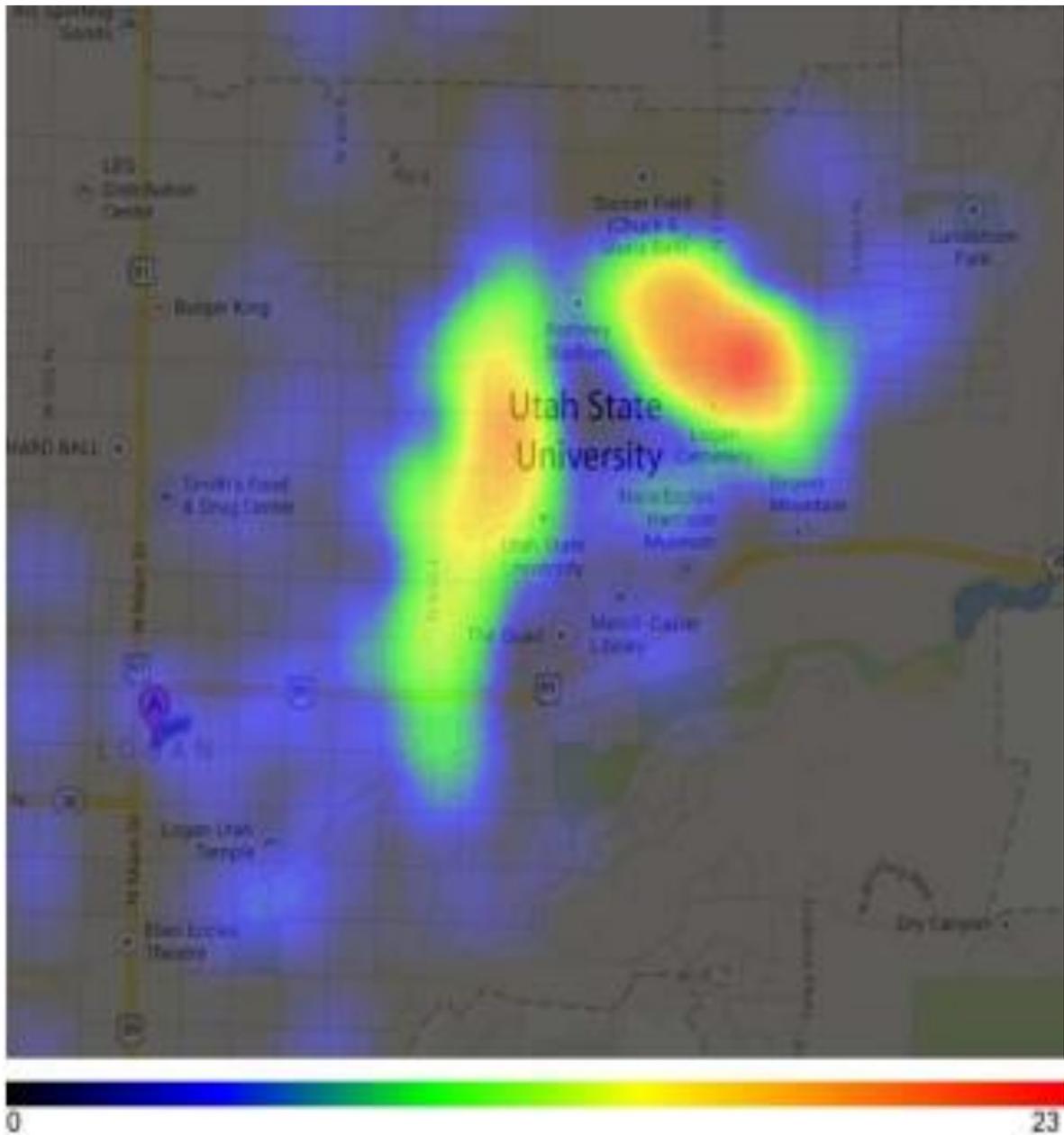


Figure 2.5 The map starting point when biking to campus

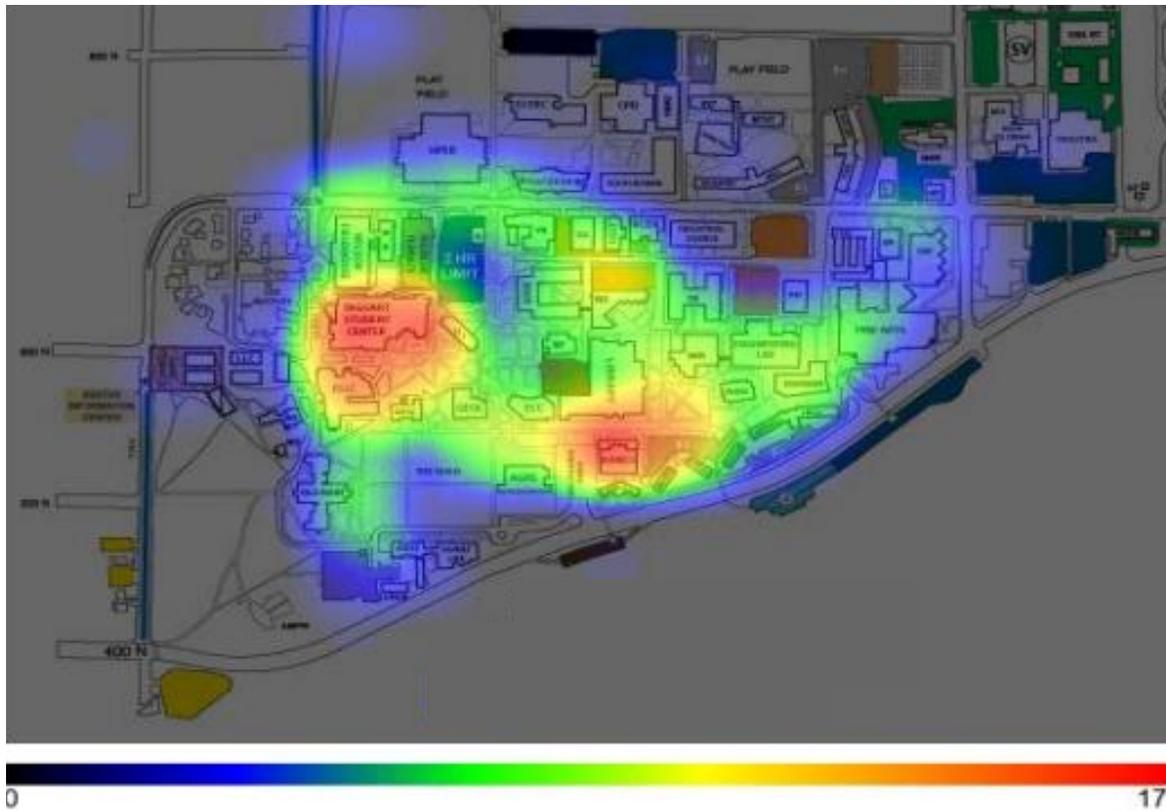


Figure 2.6 The map where more bike parking is located

2.6 Bicycle counts

The motorized mode traffic counts are regularly collected from count detectors on the street, but there is no regular collection of bicycle data (e.g., bicycle counts on streets and intersections). To obtain bicycle traffic information for the USU campus, we manually collected bicycle counts in September 2014. However, since bicycle counts are not collected regularly at the USU campus, our bicycle count data may not be an accurate representation of actual cycling activity. If there are such issues with data inconsistency between bicycle counts, it would affect the estimation of the bicycle O-D demand matrix. Figure 2.7 summarizes our bicycle count results, which were conducted on a daily basis at specific locations on campus.

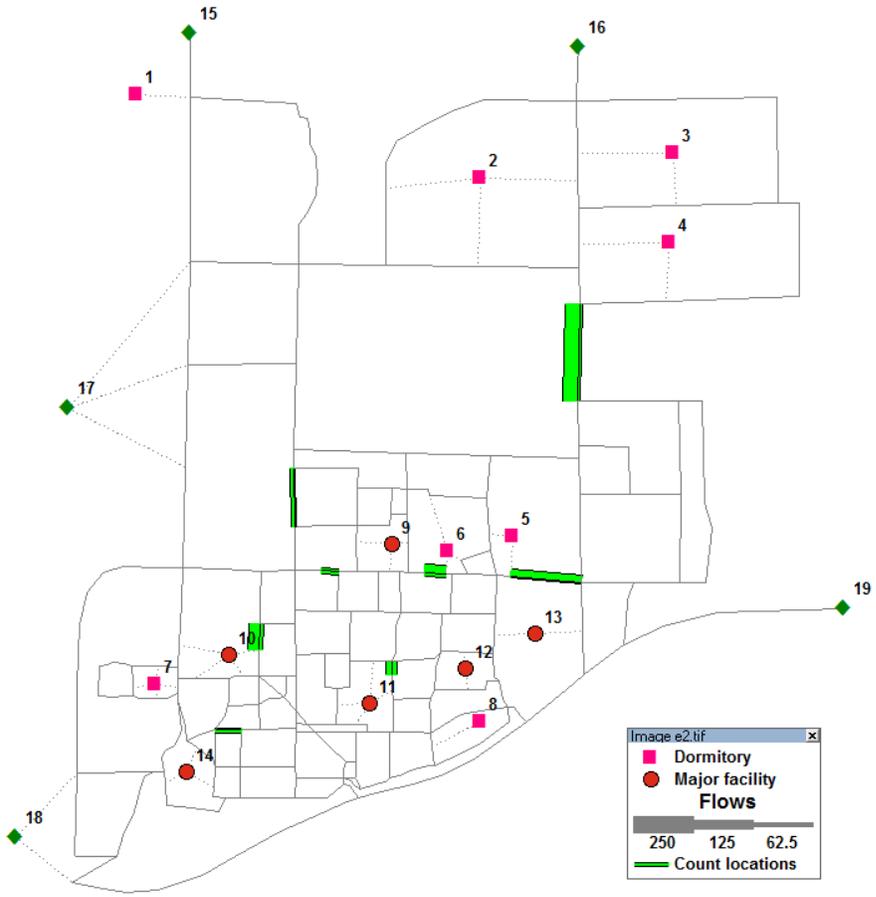


Figure 2.7 Bicycle counts and location (8 locations and 16 directional counts)

3. INITIAL BICYCLE O-D DEMAND MATRIX GENERATION

In this module, we will present the bicycle O-D demand generation procedure (O-D matrix) for the USU network using available data collected in Section 2 (specifically from Table 2.2, Figure 2.5 and Figure 2.6). The inputs to the O-D demand generation procedure include the estimated zonal production and attraction flows from the USU transportation survey data, the model choice proportion data from Table 2.2, the proportion for origin data from Figure 2.5 and the proportion for destination data from Figure 2.6. Figure 3.1 presents an overall procedure for the bicycle O-D estimation problem. First, zonal production and attraction flows are generated with obtained data from travel surveys, and then the O-D demand is generated using the doubly constrained gravity model with the bicycle commute trip length (friction factor) that is obtained by Aultman-Hall et al. (1997) as deficiency in the area.

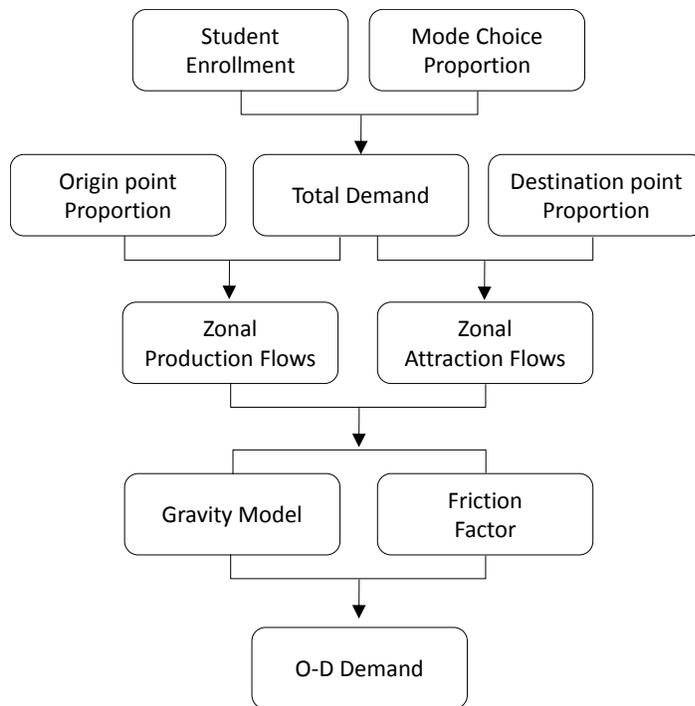


Figure 3.1 O-D demand generation procedure

3.1 Generating Zonal Production and Attraction Flows (Trip Generation)

Trip generation (or travel choice) is the first step for generating O-D demand. It predicts the number of trips originating in or destined for a particular traffic analysis zone (TAZ). There are two types of trip generation models: (a) production models and (b) attraction models. Production models estimate the number of trips generated from each TAZ, while attraction models estimate the number of trips attracted to each TAZ. Typical methods used to model trip generation include: (1) regression equations at an aggregate (zonal) or disaggregate (household) level, (2) category (or cross classification) models, and (3) ITE (Institute of Transportation Engineers) trip generation rates.

To generate the zonal production and attraction flows, we first estimate total bicycle trips using total enrollment data (14,000 students) and the mode choice probability from Table 2.2.

Step 1: Estimating total demand: total enrollment \times mode choice probability

After estimating the total bicycle trips, we can estimate the zonal production and attraction flows for each zone using the starting proportion and destination proportion from Figure 2.5 and Figure 2.6.

Step 2: Estimating zonal production flows: total demand \times starting proportion

Estimating zonal attraction flows: total demand \times destination proportion

Figure 3.2 graphically depicts the generated zonal production and attraction flows for the USU campus. Step 1 used the enrollment data and mode choice proportion data to estimate a total of 1,166 bicyclists on the university campus. This estimation of 1,166 cyclists are then distributed to each zone in Step 2.

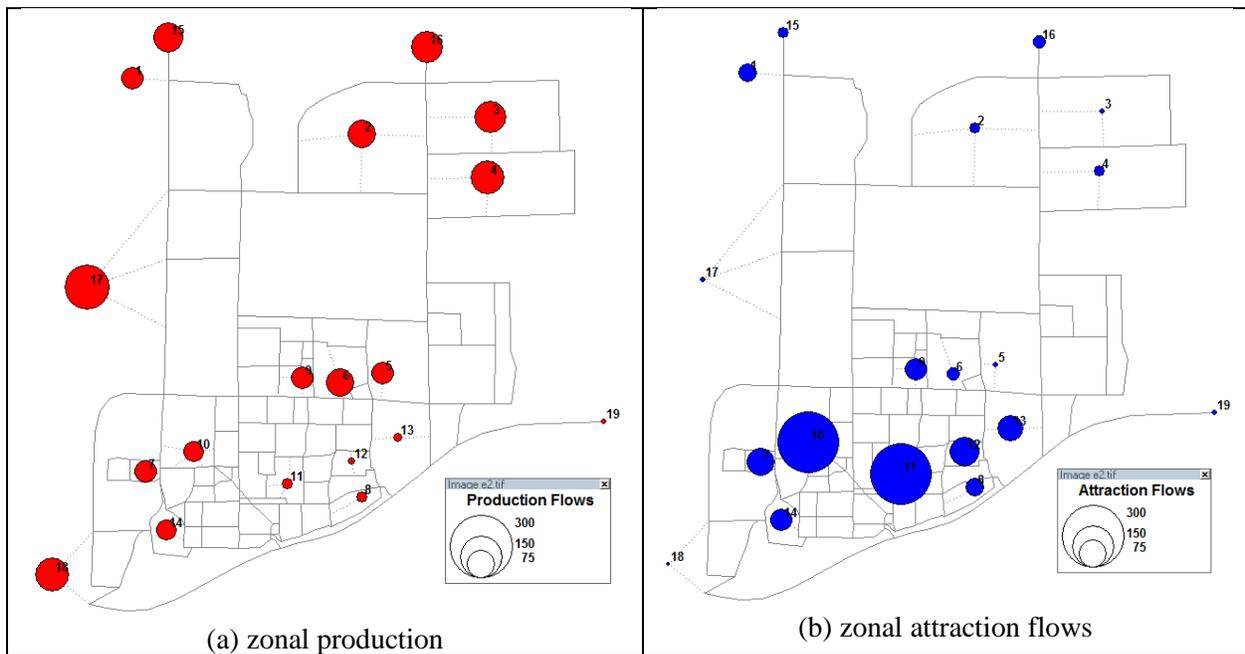


Figure 3.2 Generated zonal production and attraction flows at Utah State University

3.2 Trip Distribution (Gravity Model)

Once the trip productions and attractions for each zone are computed, the trips can be distributed among the zones using a trip distribution model. The trip distribution model is essentially a destination choice model that generates a trip table using the trip ends produced from the trip generation models and network attributes (e.g., interzonal travel times). Trip distribution has traditionally been based on the gravity model (e.g., the doubly constrained gravity model) as follows:

$$T_{ij} = \frac{P_i}{\sum_i P_i \cdot K_i \cdot F_{ij}} \cdot \frac{A_j}{\sum_j A_j \cdot K_j \cdot F_{ij}} \cdot F_{ij}, \quad (3.1)$$

where T_{ij} = Number of trips produced in zone i and attracted to zone j

P_i = Number of trips produced in zone i

A_j = Number of trips attracted to zone j

F_{ij} = An empirically derived “*friction factor*,” which expresses the average area-wide effect of spatial separation on the trip interchanges between the two zones, i and j

K_{ij} = Empirically origin and destination adjustment factor, which takes into account the effects on travel patterns of defined social and economic linkages not otherwise incorporated into the model

In the absence of friction factors in the university area, we substitute the friction factors with the bicycle commute trip length introduced by Aultman-Hall et al. (19) 97as shown in Figure 3.3.

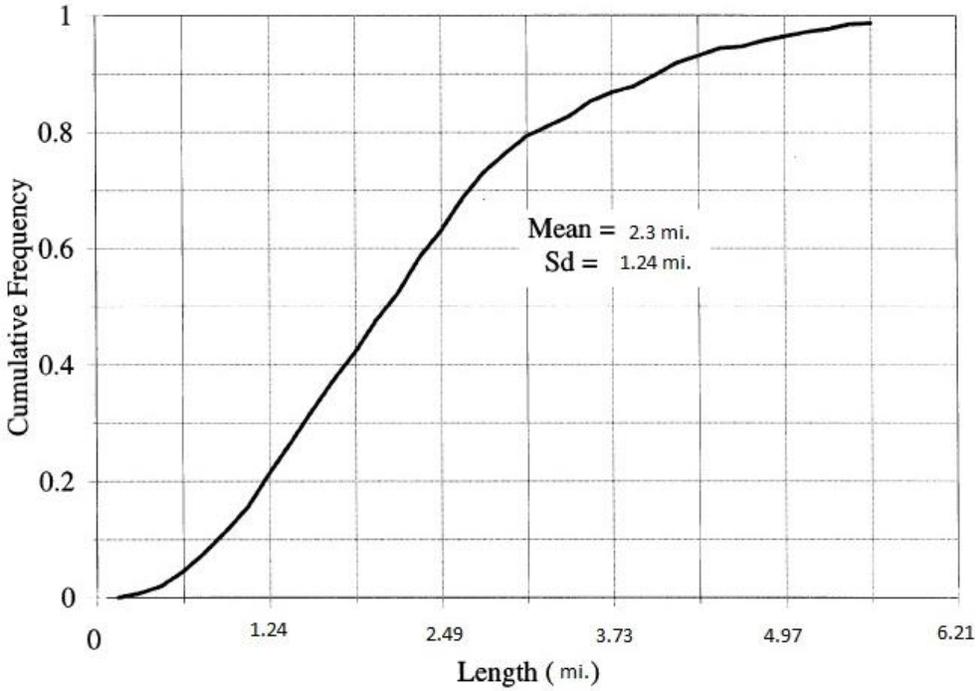


Figure 3.3 Bicycle commute trip length (Aultman-Hall et al., 1997)

Finally, the gravity model is performed with the obtained zonal production flows, attraction flows, and friction factor. Figure 3.4 presents the final generated O-D demand from the gravity model.

Step 3: Generating O-D demand with zonal production and attraction flows and friction factors

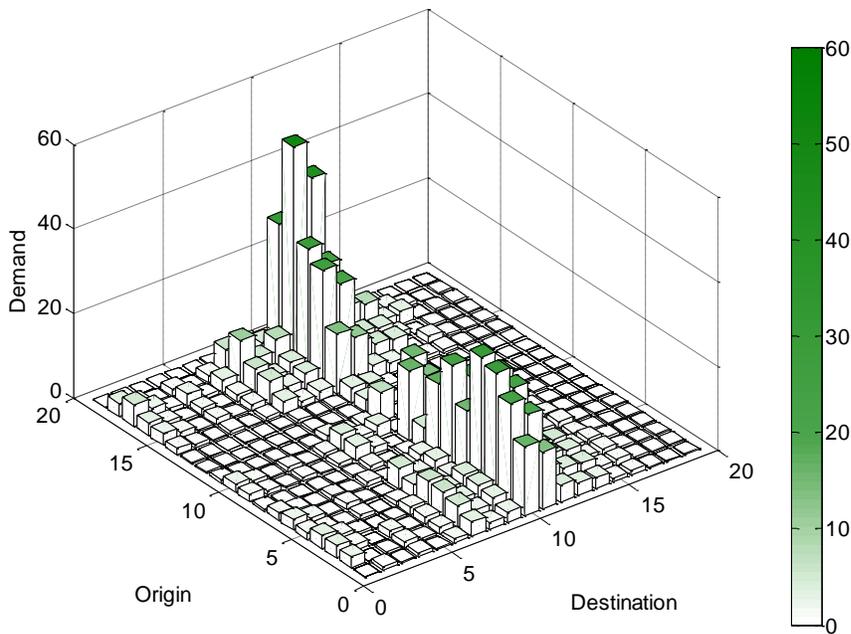


Figure 3.4 Generated O-D demand

4. A TWO-STAGE BICYCLE TRAFFIC ASSIGNMENT

In this module, we will develop a two-stage bicycle traffic assignment procedure for assigning a bicycle O-D trip table to the bicycle network to obtain the bicycle traffic flow pattern. The core component of the multi-criteria bicycle traffic assignment model is the factors (or criteria) affecting the cyclists' route choice decisions. In this research, we will explore different factors relevant to cyclists' route choice decisions (e.g., shortest distance, BLOS, BCI, BSL, etc.) to develop a multi-criteria traffic assignment model that explicitly considers each criterion as an objective in the model. We will then develop a bi-objective traffic assignment model using distance and BLOS as two distinct objectives for cyclists' route choice decisions. The overall procedure for solving the bi-objective bicycle assignment model is shown in Figure 4.1. It consists of two main steps: (1) determining efficient routes that represent the optimal tradeoffs between distance and BLOS by generating a Pareto set of routes, and (2) determining the flow allocation to each route in the cyclists' route choice set. Step 1 involves developing a bi-objective shortest path algorithm to generate the optimal routes, while Step 2 involves developing a path size logit assignment procedure for allocating the bicycle demand to the optimal routes generated in the first step.

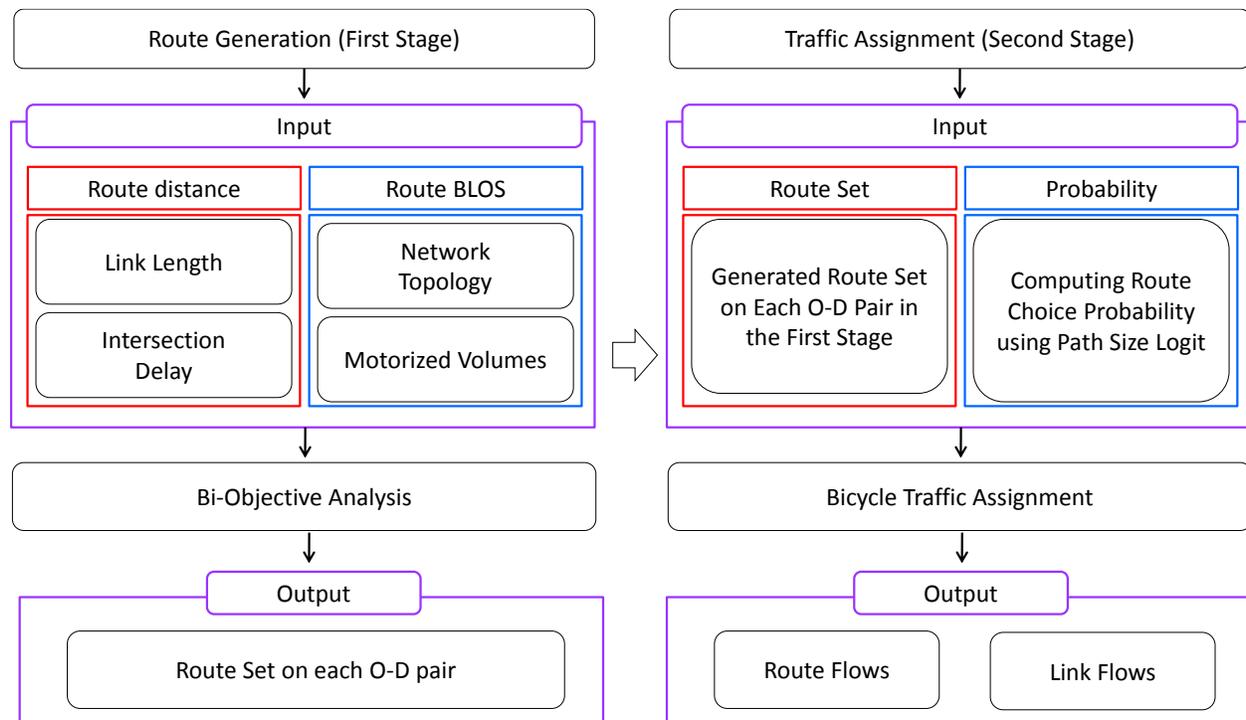


Figure 4.1 Bicycle traffic assignment procedure

4.1 Two Key Cyclist Route Choice Criteria

Models for private motorized vehicles may rely on a conventional single objective as the sole criterion for determining route choice decisions (i.e., the Wardrop's user equilibrium model is based on flow-dependent travel times), but this single objective criterion may not be adequate for bicycle route choice models due to their diverse influential factors (Menghini et al., 2010; Kang and Fricker, 2013). In this paper, we adopt two key criteria (e.g., route distance related attributes and route safety related attributes) to capture the most important factors in affecting cyclist route choice behavior.

4.1.1 Route Distance

Route distance is a composite measure of not only the sum of link distances along the route, but also of the delays at signalized intersections that the route passes through. For bicycle trips, intersection delays have been shown to be a deterrence to cyclists' route choice behavior. In order to combine these two different measurements (i.e., link length measured in km and intersection delay measured in seconds), we convert delay to an equivalent distance unit with an appropriate conversion factor (i.e., similar to the value of time used to convert time to an equivalent monetary value) as follows:

$$d_k^{rs} = \sum_{a \in k} l_a + \sum_{i \in k} cf_i^m \cdot d_i^m, \quad rs \in \text{RS}, k \in \text{K}_{rs} \quad (4-1)$$

where d_k^{rs} is the distance on route k connecting O-D pair rs ; l_a is the length on link a ; cf_i^m is the conversion factor for turning movement m at intersection i ; d_i^m is the delay of turning movement m at intersection i ; RS is the set of O-D pairs; and K_{rs} is the set of routes connecting O-D pair rs . The route distance in Eq. (4-1) can be computed by summing the link distances (first term) and intersection delays (second term) that comprise that route.

4.1.2 Route Bicycle Level of Service (BLOS)

There are numerous measures for assessing the safety aspect of bicycle facilities or the suitability for bicycle travel. Lowry et al. (2012) provided a recent review of 13 methods used in the literature. All methods attempt to provide a score of the perceived safety of bicycle facilities by using a linear regression with variables that represent the conditions of the roadway and the environment that affects a cyclists' comfort level. For this study, we adopt the bicycle level of service (BLOS) developed by the HCM (2010) as a surrogate measure to account for different attributes contributing to the safety of bicycle routes. The BLOS measure is considered as the state-of-the-art method, and has been adopted by many cities in the United States as a guide for bicycle facility design. However, other bicycle safety measures could be used in our proposed framework for modeling cyclists' route choice behavior. The route BLOS measure described in Eq. (4-2) is a composite measure based on the average segment bicycle score on a route ($ABSeg$), average intersection bicycle score on a route ($ABInt$), and average number of unsignalized conflicts/driveways per mile on a route ($Cflt$). The BLOS equation is as follows:

$$BLOS = 0.200 \cdot (ABS_{seg}) + 0.030 \cdot (\exp(ABInt)) + 0.050 \cdot (Cflt) + 1.40 \quad (4-2)$$

- $ABS_{seg} = \sum_a l_a \cdot Bseg_a / \sum_{a \in k} l_a$: the length weighted by the average segment bicycle score ($Bseg_a$) on a route
 - $ABInt = \sum_n IntBLOS_n / N_k$: average intersection bicycle score ($IntBLOS_n$) on a route
 - $Cflt$: the number of unsignalized conflicts/driveways per mile
 - l_a is the length on link a
 - N_k is the total number of intersections on route k
-

Note that the segment and intersection bicycle scores ($Bseg_a$ and $IntBLOS_n$) provided in Eqs. (4-3) and (4-4) are calibrated based on the volume and speed of motorized vehicles, the width configuration of bicycle facilities, pavement conditions, number of intersections, etc. The derived BLOS score is a relative measurement without score units to evaluate the comfort, convenience, or utility of the cycling route. The details of the BLOS development can be found in NCHRP Report 616 (Dowling et al., 2008).

$$BSeg = 0.507 \ln\left(\frac{V}{4 \cdot PHF \cdot L}\right) + 0.199 F_s (1 + 10.38 \cdot HV)^2 + 7.066 \left(\frac{1}{PC}\right)^2 - 0.005(We)^2 + 0.76 \quad (4-3)$$

- | | |
|---|---|
| <ul style="list-style-type: none"> • PHF : peak hour factor • HV : proportion of heavy vehicles in motorized vehicle volume • We : average effective width of outside through lane (ft) | <ul style="list-style-type: none"> • F_s : effective speed factor • L : total number of directional through lanes • V : directional motorized vehicle volume (vph) • PC: <i>FHWA</i>'s five point pavement surface condition rating |
|---|---|
-

$$IntBLOS = -0.2144 \cdot Wt + 0.0153 \cdot CD + 0.0066 \left(\frac{Vol15}{L}\right) + 4.1324 \quad (4-4)$$

- | | |
|--|--|
| <ul style="list-style-type: none"> • Wt: width of outside through lane plus paved shoulder (including bike lane where present) | <ul style="list-style-type: none"> • CD: crossing distance, the width of the side street (including auxiliary lanes and median) • $Vol15$: volume of directional traffic during a 15 minute period |
|--|--|
-

4.2 Stage One: Bi-objective Shortest Path Procedure

Solving the bi-objective shortest path problem is similar to solving any multi-objective optimization problem; in either case, a single optimal solution may not exist that dominates all other solutions in all objectives. Hence, solving multi-objective problems requires generating a set of non-dominated (or Pareto) solutions. The bi-objective shortest path problem belongs to a class of NP-hard problems (Serafini, 1986). Several solution procedures have been developed to solve this complex problem. These solution procedures include the label correcting approach (Skriver and Andersen, 2000), the label setting approach (Tung and Chew, 1992), the ranking method (Climaco and Martins, 1982), and the two-phase method (Ulungu and Teghem, 1995). However, handling the non-additive route cost structure (e.g., route BLOS) may not be easy in these methods. In this paper, we adopt the two-phase procedure used in Ehrgott et al. (2012) to solve the bi-objective shortest problem with non-additive route cost structure. The overall two-phase procedure is described in Figure 4.2. In the first phase, it uses the distance-related attributes (i.e., link distance and intersection delay) to generate a set of realistic routes without exceeding the maximum allowable bound. In the second phase, the corresponding safety-related attributes are computed for each route in the set to determine the efficient routes according to the two key criteria: route distance and route BLOS.

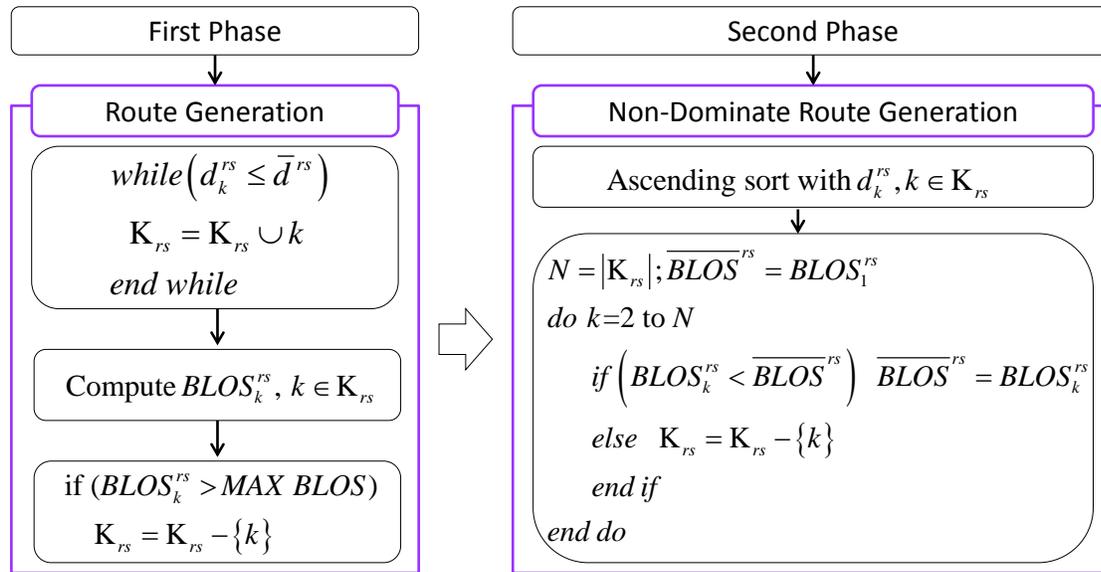


Figure 4.2 Two-phase procedure for generating non-dominant routes

4.3 Stage Two: Bicycle Traffic Assignment Method

The conventional bi-objective traffic assignment model was introduced by Dial (1979) for addressing the multiclass traffic assignment problem. Dial (1979, 1996, 1997) adopted a linear value of time (VOT) function to convert travel time to equivalent monetary unit, while Gabriel and Bernstein (1997) introduced a non-linear VOT function for the non-additive traffic equilibrium problem. Nagurney (2000), Nagurney et al. (2001, 2002), and Nagurney and Dong (2002) proposed variable weights for the multi-criteria traffic assignment problem by assuming a linear generalized cost function for combining the criteria with variable weights. In this paper, we adopt a path-size logit (PSL) traffic assignment method, which has been widely adopted in the literature as a multipath traffic assignment method (Menghini et al. 2010; Hood et al. 2011).

4.3.1 Path-size Logit Assignment (PSLA) Method

The PSLA method assigns the O-D demand based on the combined utilities of two objectives via the path size logit choice function. The multinomial logit (MNL) model is a widely used route choice model under the random utility principle. However, it is well known that the major drawback in applying the MNL model to the route choice problem is the inability to account for overlapping (or correlation) among routes. Ben-Akiva and Bierlaire (1999) proposed the path size logit model (PSL) as an alternative to solve the overlapping problem in MNL. The closed-form probability of PSL is expressed as follows:

$$P_k^{rs} = \frac{PS_k^{rs} \cdot \exp(U_k^{rs})}{\sum_{j=1}^n PS_j^{rs} \cdot \exp(U_j^{rs})}, \quad \forall k \in K_{rs}, rs \in RS$$

$$\text{where, } U_k^{rs} = -\left((d_k^{rs})^\alpha \cdot (BLOS_k^{rs})^\beta \right), \quad \forall k \in K_{rs}, rs \in RS, \quad (4-5)$$

$$PS_k^{rs} = \sum_{a \in k} \left(\frac{l_a}{L_k^{rs}} \right) \cdot \left(\frac{1}{\sum_{l \in K_{rs}} \delta_{la}^{rs}} \right), \quad \forall k \in K_{rs}, rs \in RS$$

U_k^{rs} is the utility of route k between O-D pair rs ; d_k^{rs} is the distance of route k between O-D pair rs ; and $BLOS_k^{rs}$ is the bicycle level of service of route k between O-D pair rs ; α and β are parameters; L_k^{rs} is path length on path k between origin r and destination s ; and l_a is the length of link a .

4.4 Numerical Study of Bicycle Traffic Assignment in USU Campus

In this section, the two-stage approach is applied to a university campus, Utah State University (USU), in Logan, Utah. Figure 4.3 depicts the USU campus, its bicycle network and its O-D demands. The network consists of 19 zones, 714 links, and 342 O-D pairs. Of the 714 links, 406 links are the non-motorized links (i.e., no motorized vehicles) and 74 links are shared links for both bicycles and motorized vehicles.

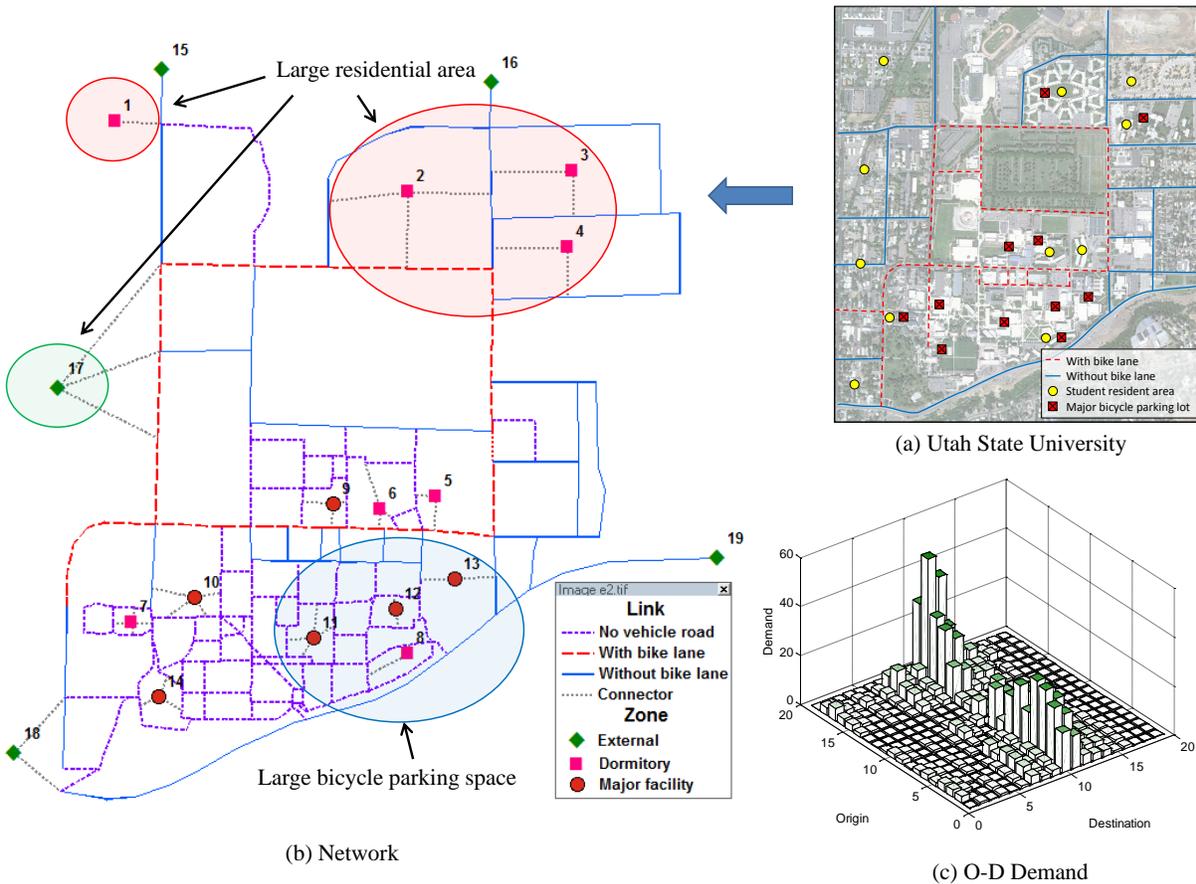


Figure 4.3 USU campus, its bicycle network, and O-D demand

4.4.1 Stage One: Bicycle BLOS Analysis and Route Generation Results

Figure 4.4(a) shows the estimated BLOS results in the USU network. To compute the BLOS measures with Eqs. (4-3) and (4-4), traffic conditions (e.g., motorized vehicle volumes) and space availability (e.g., lane width) are obtained from the Cache Metropolitan Planning Organization (CMPO) planning model (CMPO, 2007). A segment with a high motorized vehicle volume typically gives a higher BLOS value, while links with only non-motorized vehicles typically give a lower BLOS value. After the BLOS analysis, the two-phase route generation procedure is performed to generate the efficient routes in terms of route distance and route BLOS for each O-D pair. For the network, we assumed that the maximum distance of the route could not exceed 100 meters more than the shortest route between a O-D pair. The route distribution is

shown in Figure 4.4(b). Of the O-D pairs, 6% have only one route, about 33% have two routes, and about 18% have three routes in the USU network. Figure 4.4 (c) illustrates an example of Pareto frontier for O-D pair 1-11.

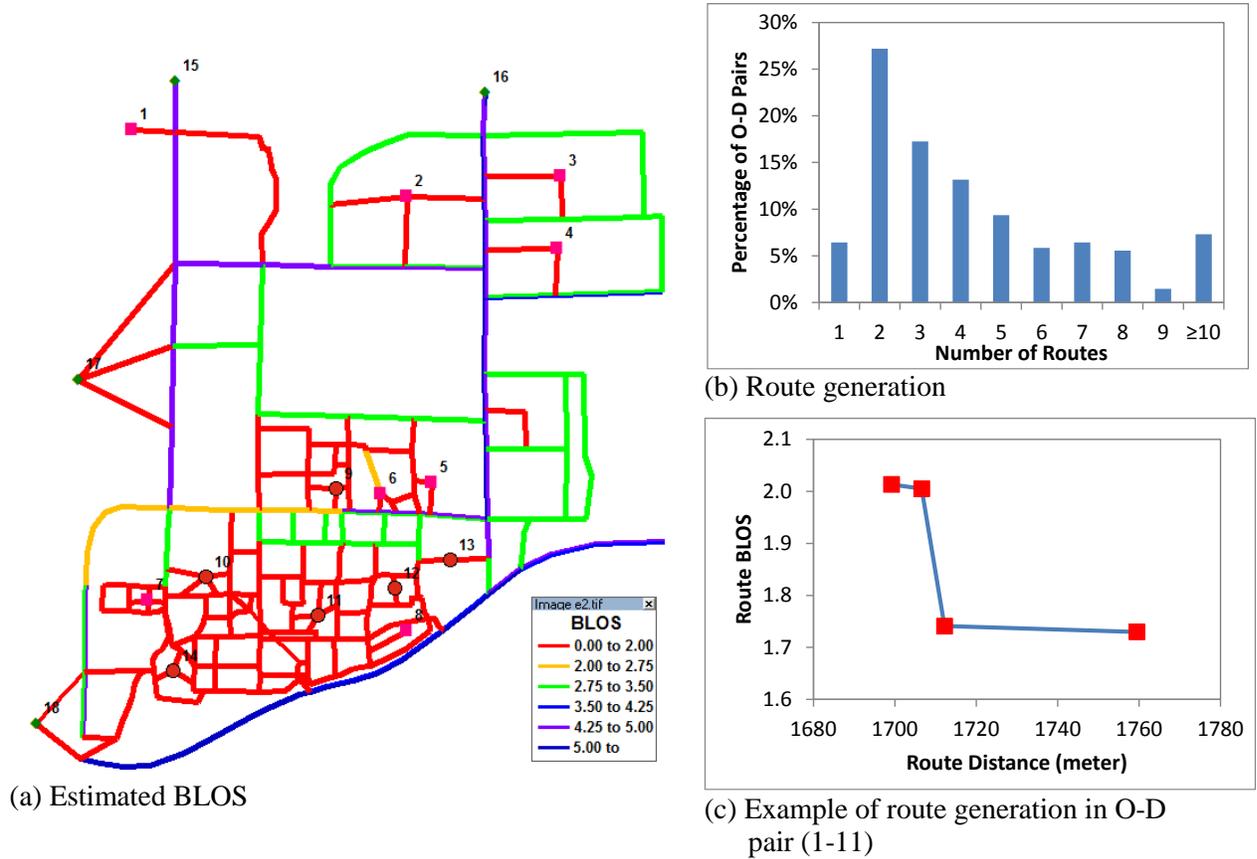


Figure 4.4 Network BLOS and generated routes

4.4.2 Stage Two: Bicycle Traffic Assignment Results

Using the generated non-dominated routes from the first stage, we perform the bicycle traffic assignment procedure. For the utility function, we adopted a composite utility function of route distance multiplied by route BLOS, $U_k^{rs} = -\left(\left(d_k^{rs}\right)^\alpha \cdot \left(BLOS_k^{rs}\right)^\beta\right)$ with parameters, $\alpha = 0.862$ and $\beta = 0.117$, suggested by Kang and Fricker (2013).

Figure 4.5(a) depicts the assigned link flow patterns. As can be seen, two major roads are used to travel from the north area (dormitories) to the south area (university). However, the right major road has more observed flows in Figure 2.7. Because the used O-D demand is generated by the assumed friction factors obtained from Aultman-Hall et al. (1997), the assigned flows are different compared with observed flows. Figure 4.5(b) provides a detailed comparison between the assigned and observed flows; it shows the scatter plot between observed counts in Figure 2.7 and estimated flows from the assignment. We can observe that the flow in many locations (link) have out of error bounds (e.g., 20%). Hence, the O-D demand is required for the calibration process. The O-D demand adjustment will be discussed in Section 5. Figure 4.5(c) shows the

distribution of the assigned link flows. As the bicycle demand is relatively smaller than motorized demand, about 47% of links have less than five flows, while 17% of links have more than 50 flows.

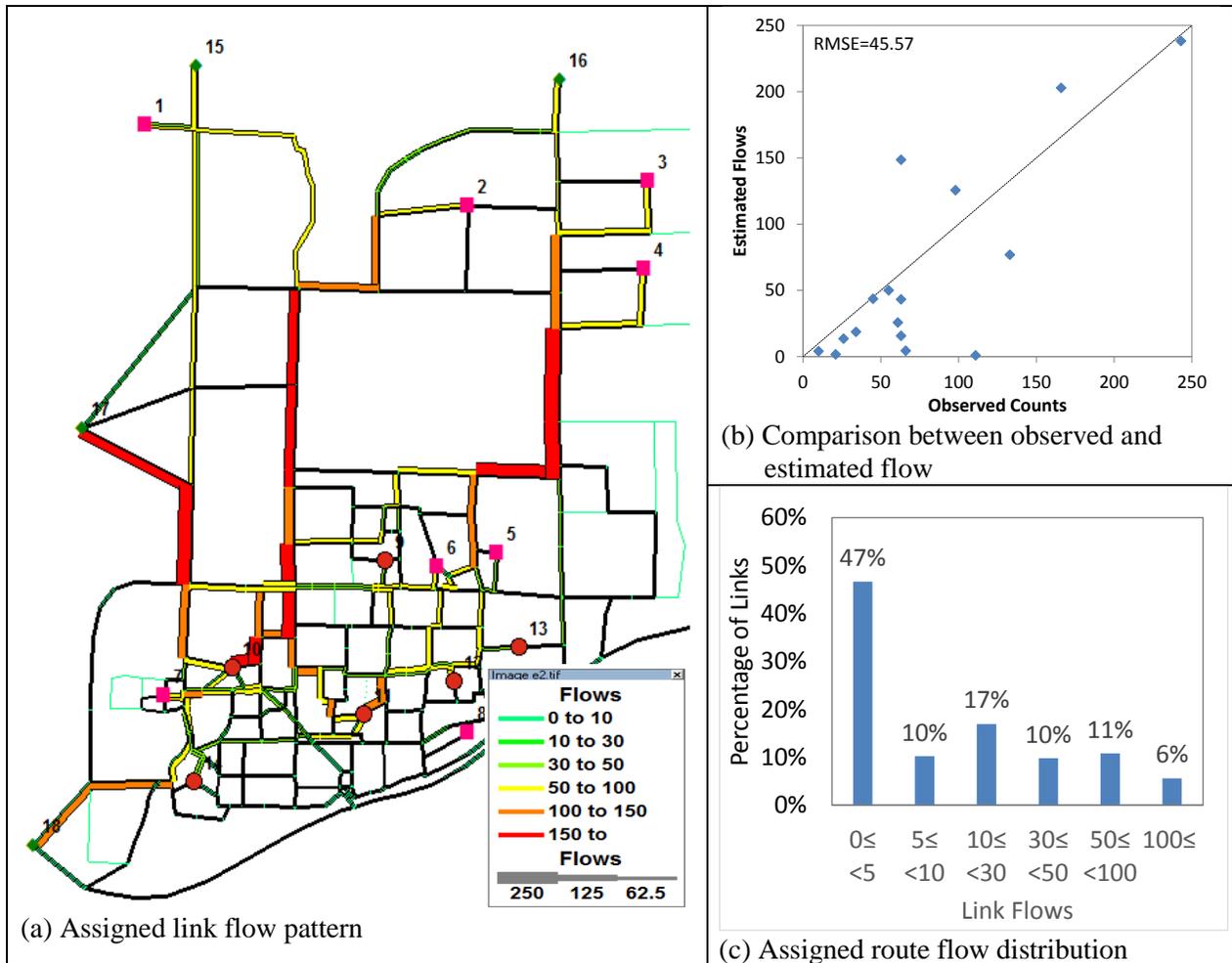


Figure 4.5 Link flow patterns

5. BICYCLE O-D DEMAND ADJUSTMENT PROCEDURE

The assigned O-D demand in Section 4 is estimated based on the trip distribution model (the doubly constrained gravity model from Section 3), so it requires a calibration process. However, modelers are often forced to skip the calibration of trip distribution models due to the unavailability of data on Trip Length Frequency Distribution (TLFD) of local travelers. Thus, calibration and validation of the overall model are often carried out by altering the friction factors and by adding k-factors in a trial-and-error fashion to the trip distribution model. This process ensures that the results of traffic assignment would match traffic counts on selective screenlines and critical links. The calibration process is usually lengthy and the resultant models often contain many factors that do not have the necessary behavioral foundation established from travel surveys.

For this project, we also adopted the two-stage bicycle O-D demand adjustment procedure developed by Ryu et al. (2015). Stage 1 is responsible for the determination of efficient (or non-dominated) routes that represent the optimal tradeoffs between route distance and route bicycle level of service (BLOS) by generating a Pareto set of routes (i.e., the same route set adopted in the bicycle traffic assignment procedure). Stage 2 is responsible for the adjustment of O-D demand based on observed link counts and obtained zonal production flows, zonal attraction flows, and initial O-D demand estimated in Section 3. It uses a path flow estimator (PFE) (Bell and Shield, 1995; Bell et al., 1997; Chen et al., 2005, 2009, 2010; Chootinan et al., 2005) to refine the initial bicycle O-D matrix from Section 3 such that the readjusted, final bicycle O-D matrix can reproduce better matches with the observed bicycle counts when performing the bicycle traffic assignment procedure. The flexibility of aggregating path flows at different spatial levels, which allows the usage of various data (e.g., bicycle intersection counts, bicycle link counts, bicycle GPS data, bicycle miles traveled [BMT], etc.), makes the PFE a suitable approach for improving the accuracy of bicycle O-D estimation. Figure 5.1 presents an overall procedure for the bicycle O-D estimation problem.

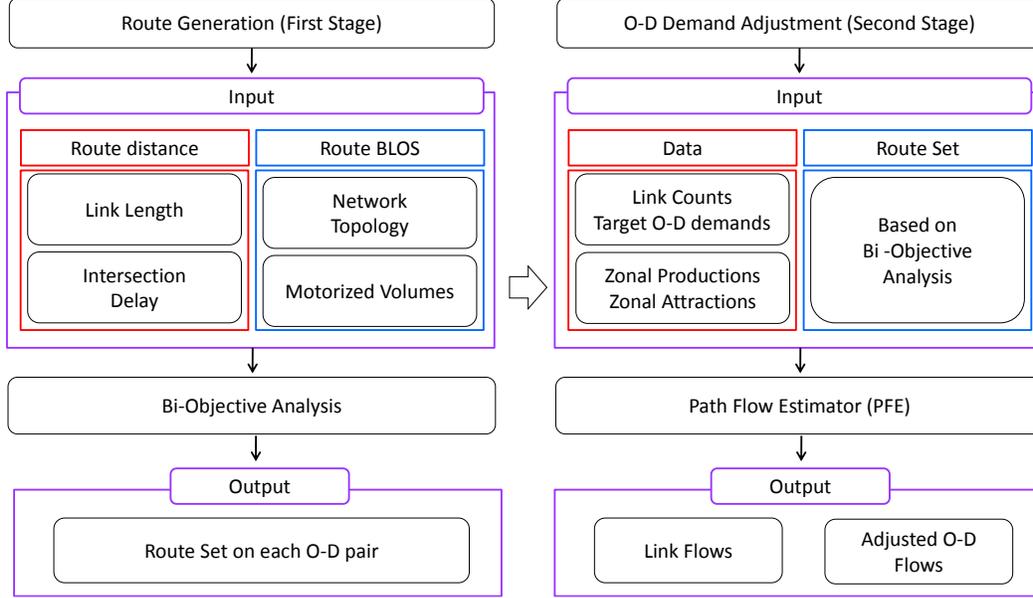


Figure 5.1 Bicycle O-D demand adjustment procedure

5.1 Path Flow Estimator (PFE)

The nonlinear Path Flow Estimator (PFE) was originally developed by Bell and Shields (1995) as a one-stage network observer. It is able to estimate path flows and path travel times using incomplete traffic data collected in the field. The core component of PFE is a logit-based path choice model in which the perception errors of path travel times are assumed to be independent Gumbel variates (Dial, 1971). The logit model interacts with link cost functions to produce a stochastic user equilibrium (SUE) traffic pattern. The aim of this section is to adapt the PSL-based PFE to take not only field data (e.g., traffic counts) but also planning data (e.g., zonal production and attraction flows and target O-D trip table) to adjust the O-D trip table estimated by the gravity model in Section 3. The PSL-based PFE formulation can be formulated with route utilities as a convex program with various side constraints as follows:

$$\text{Minimize: } Z(\mathbf{f}) = \sum_{rs \in \mathbf{RS}} \sum_{k \in \mathbf{K}_{rs}} f_k^{rs} (\ln f_k^{rs} - 1) - \sum_{rs \in \mathbf{RS}} \sum_{k \in \mathbf{K}_{rs}} U_k^{rs} f_k^{rs} - \sum_{rs \in \mathbf{RS}} \sum_{k \in \mathbf{K}_{rs}} f_k^{rs} \ln PS_k^{rs} \quad (5-1)$$

$$\text{subject to: } (1 - \varepsilon_a) \cdot v_a \leq x_a \leq (1 + \varepsilon_a) \cdot v_a, \quad \forall a \in \bar{\mathbf{A}}, \quad (5-2)$$

$$(1 - \varepsilon_r) \cdot O_r \leq P_r \leq (1 + \varepsilon_r) \cdot O_r, \quad \forall r \in \bar{\mathbf{R}}, \quad (5-3)$$

$$(1 - \varepsilon_s) \cdot D_s \leq A_s \leq (1 + \varepsilon_s) \cdot D_s, \quad \forall s \in \bar{\mathbf{S}}, \quad (5-4)$$

$$(1 - \varepsilon_{rs}) \cdot z_{rs} \leq q_{rs} \leq (1 + \varepsilon_{rs}) \cdot z_{rs}, \quad \forall rs \in \bar{\mathbf{RS}}, \quad (5-5)$$

$$f_k^{rs} \geq 0, \quad \forall k \in \mathbf{K}_{rs}, rs \in \mathbf{RS}, \quad (5-6)$$

where

$$x_a = \sum_{rs \in \text{RS}} \sum_{k \in \text{K}_{rs}} f_k^{rs} \delta_{ka}^{rs}, \quad \forall a \in \text{A}, \quad (5-7)$$

$$P_r = \sum_{s \in \text{S}} \sum_{k \in \text{K}_{rs}} f_k^{rs}, \quad \forall r \in \text{R}, \quad (5-8)$$

$$A_s = \sum_{r \in \text{R}} \sum_{k \in \text{K}_{rs}} f_k^{rs}, \quad \forall s \in \text{S}, \quad (5-9)$$

$$q_{rs} = \sum_{k \in \text{K}_{rs}} f_k^{rs}, \quad \forall rs \in \text{RS}, \quad (5-10)$$

where f_k^{rs} is the flow on route k connecting O-D pair rs ; U_k^{rs} is the utility route k between origin r and destination s ; PS_k^{rs} is the path size factor on route k between origin r and destination s ; v_a is the observed count on link a ; x_a is the estimated flow on link a ; ε_a is the percentage of measurement error allowed for the traffic count on link a ; $\bar{\text{A}}$ is the set of network links with measurements; O_r and D_s are the generated trip production of origin r and generated trip attraction of destination s obtained by the 2014 Transportation Survey (Utah State University, 2014) in Section 2; P_r and A_s are the estimated trip production of origin r and estimated trip attraction of destination s ; ε_r and ε_s are the error bounds allowed for trip production of origin r and trip attraction of destination s ; and $\bar{\text{R}}$ and $\bar{\text{S}}$ are the sets of zones with planning data; z_{rs} is the target O-D flows between origin r and destination s ; q_{rs} is the estimated O-D flows between origin r and destination s ; ε_{rs} is the percentage measurement error allowed for the target O-D demands between origin r and destination s ; $\bar{\text{RS}}$ is the set of target (or prior) O-D pairs; and δ_{ka}^{rs} is the path-link indicator, 1 if link a is on path k between O-D pair rs and 0 otherwise.

The objective function Eq. (5-1) has three terms: an entropy term and a system optimization term. The entropy term seeks to spread trips onto multiple routes according to the dispersion parameter, while the system optimization term tends to cluster trips on the minimizing of their trips. PS value is added to consider route overlapping in the third term. As opposed to the traditional logit-based SUE model, the PFE finds route flows that minimize the PSL base objective function in Eq. (5-1) while simultaneously reproducing traffic counts on all observed links in Eq. (5-2), zonal production and attraction of certain origin and destination in Eqs. (5-3) and (5-4), and target travel demands of certain O-D pairs in Eq. (5-5) within some predefined error bounds. These error bounds are essentially confidence levels of the observed data at different spatial levels used to constrain the path flow estimation. More reliable data will use a smaller error bound (or tolerance) to constrain the estimated flow within a narrower range, while less reliable data will use a larger tolerance to allow for a larger range of the estimated flow. Eq. (6-6) constrains the path flows to be non-negative. Eqs. (5-7), (5-8), (5-9), and (5-10) are definitional constraints that sum up the estimated path flows to obtain the O-D demand, link flows, zonal production flows, zonal attraction flows, and O-D demands, respectively. Details

of the bicycle O-D demand adjustment procedure (i.e., optimality conditions and solution algorithm) are provided in Ryu et al. (2015).

5.2 USU Campus Case Study

Using generated non-dominated routes in Section 4, obtained bicycle counts from Figure 2.7, zonal production and attraction flows from Figure 3.2, and target O-D demand from Figure 3.4, bicycle PFE is performed for O-D demand adjustment. For the utility function, the same parameters used in PSLA are adopted. As the bicycle observed counts and flows (e.g., zonal production flow, zonal attraction flow, and O-D demand) are relatively smaller than motorized volumes, there are limitations to apply the uniform error bounds. For the project, we adopt the classified error bound with different observed counts and flows as follows:

Table 5.1 Adopted error bound

Link Count	Zonal production and Attraction		Target O-D demand	
	Flows	Error bound	Demand	Error bound
30%	$0 \leq$ and <10	50%	$0 \leq$ and <5	-
	$10 \leq$ and <30	40%	$5 \leq$ and <10	40%
	$30 \leq$ and <50	30%	$10 \leq$ and <30	30%
	$50 \leq$	20%	$30 \leq$	20%

5.2.1 Comparison of Analysis Results

After conducting the numerical study, we ran three key comparisons of the results to demonstrate the value of refining the initial bicycle demand matrix. The three comparisons are as follows: (1) link flow differences between observed and estimated data, (2) link flow differences generated from the initial and adjusted matrices, and (3) estimated O-D demand differences generated from the gravity model and the PFE.

5.2.2 Flow Comparison between Observed Counts and Estimated Flows

In the first comparison, Figure 5.2 shows the scatter plots of observed and estimated link flows in (a), zonal production flows in (b), zonal attraction flows in (c), and target O-D demand in (d). To measure the accuracy of estimated flows, the RMSE value is adopted.

As can be seen in Figures 5.2 (a), (b), (c), and (d), estimated link flows, zonal production flows, zonal attraction flows, and O-D demand well match the observed values with satisfactory error bounds. Specifically, the RMSE values for estimated link flows, zonal production flows, zonal attraction flows, and O-D demand are 18.57, 15.47, 22.60, and 4.12, respectively. The point that is out of the error bound in Figure (b) is the university library. Due to the nature of libraries'

function, the USU library generates higher trips for not only attraction flows but also production flows. Moreover, the travel survey (i.e., Figure 2.5) only surveyed the original starting point, so there is a data inconsistency problem between zonal production flows and observed link counts.

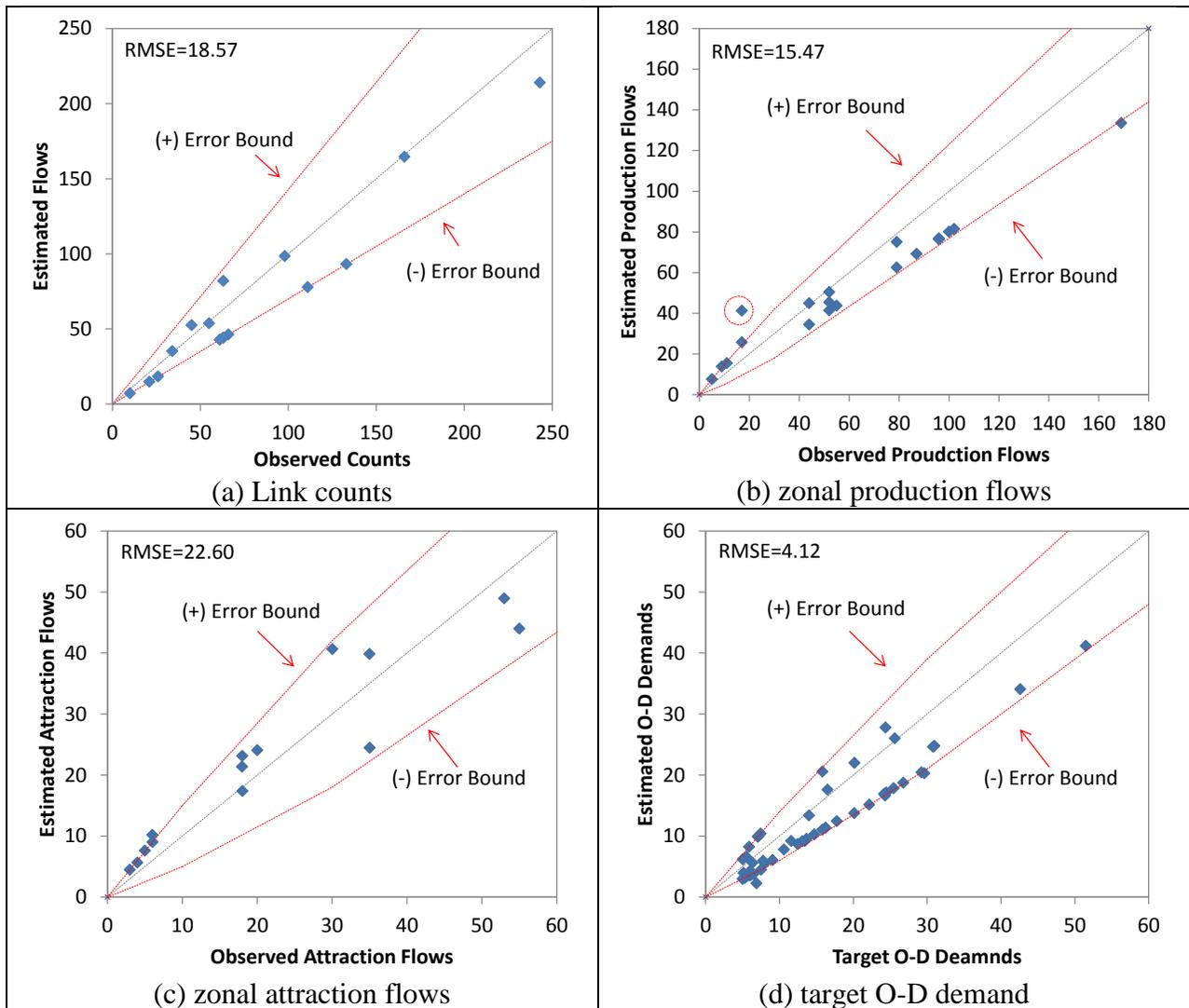


Figure 5.2 Flow comparison between observed and estimated

5.2.3 Link Flow Comparison between Assignment and PFE

In the second comparison, we look at the link flow differences between traffic assignment results generated by the initial matrix and the PFE results generated by the adjusted matrix. The initial matrix uses the PSLA model, and the adjusted matrix uses the PFE model. Figure 5.3 provides the flow allocation comparison between the PSLA and PFE models. Figure 5.3(a) depicts the flow difference on a color-coded GIS map. Red indicates that the link flows by the PFE model have higher flows than the PSLA model. The light blue (or cyan) color indicates the reverse; it indicates that the link flows by PSLA have higher flows than the PFE model. As can be seen, the left side links of the figure have more assigned flows by PSLA, while the flows by the PFE model have more flows on right side links due to the observed bicycle counts. Figure 5.3(b)

shows the scatter plot of estimated flows by both models. The patterns may seem similar at first, but they are significantly different because bicycle flows are small. The RMSE value is 14.40. Finally, Figure 5.3(c) presents the absolute link flows difference. Only 36% (0-1) of links have similar flow patterns, while about 11% (i.e., greater than 20 flows) of links have significantly different flow patterns.

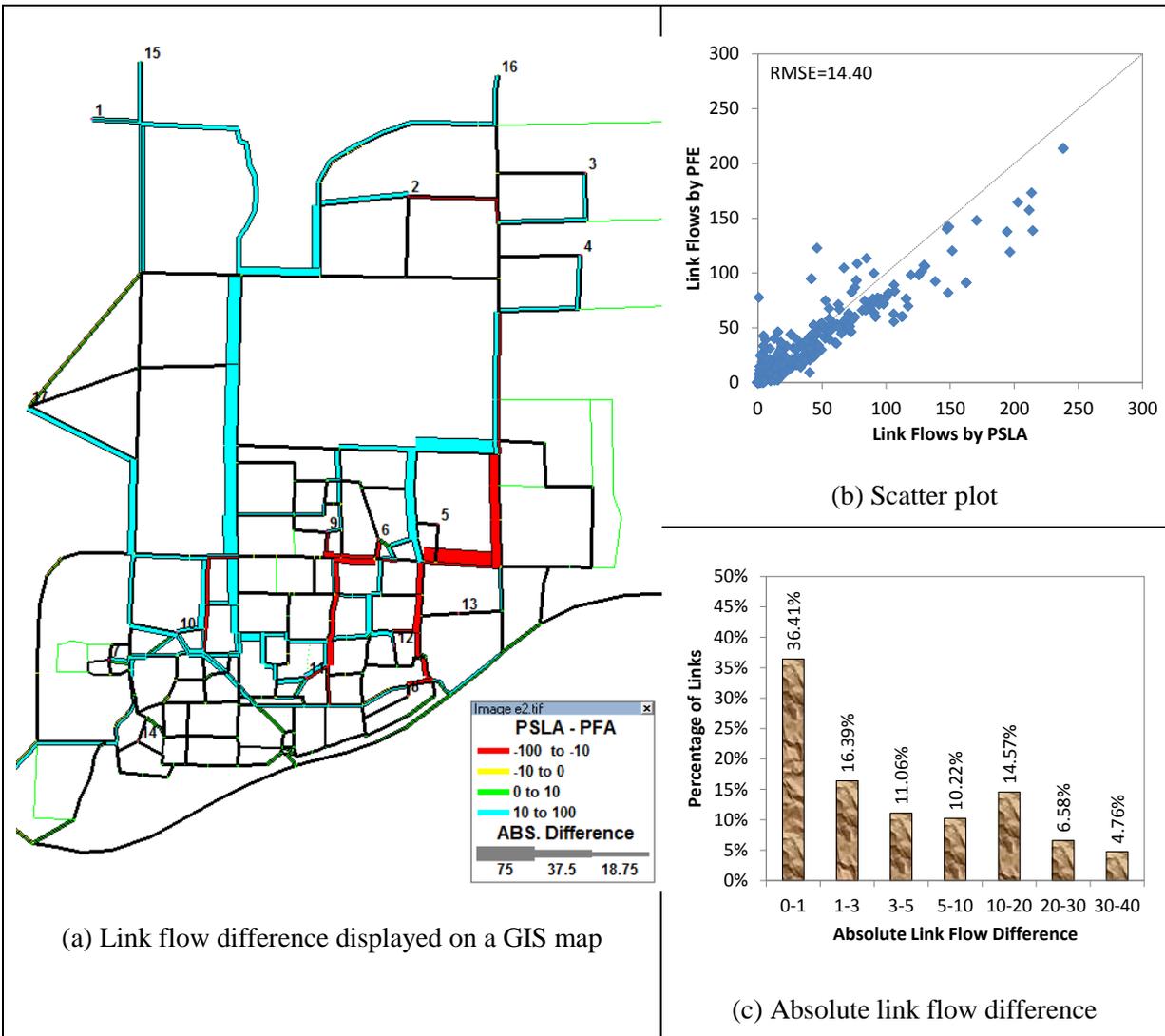


Figure 5.3 Link Flow comparison between using PSLA and PFE

5.2.4 Estimated O-D Demand Comparison between Gravity Model (GM) and PFE

In the third and final comparison, Figure 5.4 shows the adjusted O-D demand by PFE in (b) and compares it with the generated O-D by gravity model in (a) and (c). Based on Figure 5.4(b), the overall adjusted O-D demand is not significantly different compared with Figure 3.4. The RMSE value is 2.46 in Figure 5.4(c). However, we can observe that some O-D pairs were significantly affected, with an approximate difference of 10 flows in Figure 5.4(a). Basically, the O-D pairs related to the student center (zone number 10) and the library (zone number 11) have notably different demand patterns. Their demand patterns are different because these two zones have

relatively higher demand, and the two observed count locations (four directional counts) are directly related to these zones.

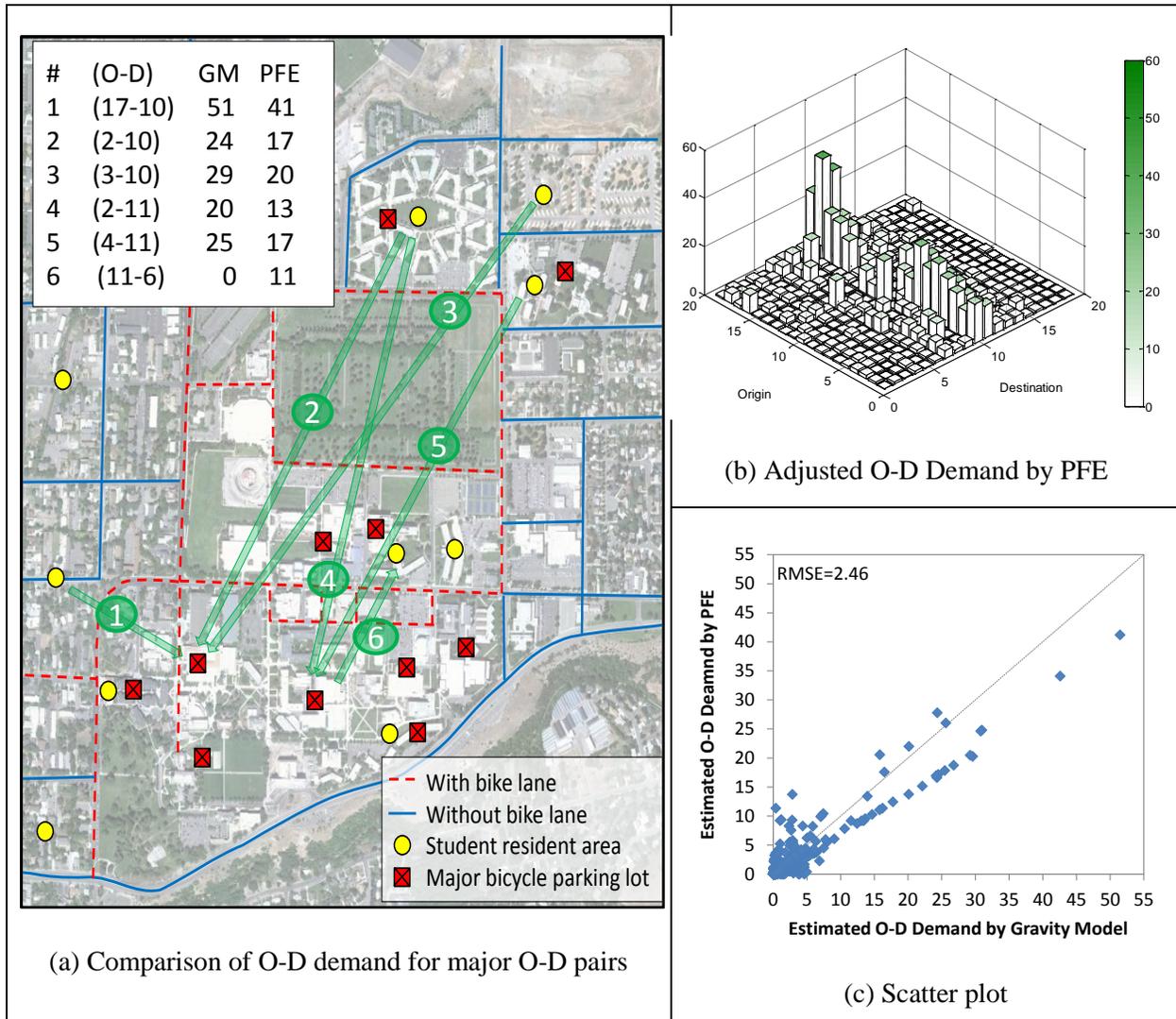


Figure 5.4 Comparison O-D demand between using gravity model and PFE

6. CONCLUDING REMARKS

This study has developed bicycle network analysis tools that can perform the following functions: initial bicycle O-D demand generation, bicycle traffic assignment, and bicycle O-D demand adjustment. Each of the analysis tools follows a particular procedure and serves a specific purpose. The initial bicycle O-D generation procedure adopts the doubly constrained gravity model to estimate an initial bicycle demand matrix. The bicycle traffic assignment procedure would then allocate the initial bicycle O-D matrix to the bicycle network to obtain the bicycle traffic flow pattern. Finally, the bicycle O-D matrix adjustment procedure refines the initial bicycle O-D matrix such that the readjusted, final bicycle O-D matrix can reproduce better matches with the observed bicycle counts when performing the bicycle traffic assignment procedure.

In terms of bicycle traffic assignment, we proposed a two-stage bicycle traffic assignment model with consideration of cyclists' route choice behavior. In the first stage of the bicycle traffic assignment procedure, we considered two key criteria, route distance and route BLOS, to generate a set of non-dominated (or efficient) paths using a bi-objective shortest path procedure. In the second stage of the procedure, we adopted a path-size logit-based assignment (PSLA) model for flow allocations to the set of efficient paths identified in the first stage. For the O-D demand adjustment, a two-stage-based model is also proposed. In the same manner with the traffic assignment model, efficient routes are generated in the first stage, and the O-D demands are adjusted with observed counts and flows in the path flow estimator.

From the traffic assignment in the Utah State University (USU) case study, we discovered that the assigned flows had values outside of the defined error bounds when compared with the observed link counts. Because the generated initial O-D demand in Section 3 assumed certain friction factors, the resulting assigned flows are different when compared with the observed counts. From the O-D demand adjustment procedure, we observed that some link flow patterns were significantly different when the observed counts were incorporated. The procedure results showed different link flow patterns because the adopted parameters in the trip distribution model and the traffic assignment model were not calibrated for the area. In addition, the USU travel survey only considered the original starting point for bicycle trips, so there was an inconsistency data problem between zonal flows and observed counts.

In this study, we chose to use the HCM's bicycle level of service (BLOS) as a surrogate measure for modeling cyclists' perception of safety (or risk) on different bicycle facility types. It would be interesting to consider other measures, such as the bicycle compatibility index (Harkey et al., 1998) or the stress indicator (Mekuria et al., 2012), and examine their impact on efficient route generation and flow allocations to the bicycle network.

Future research on this topic should consider the following four suggestions: (1) conducting more tests with different network topologies with different bicycle facilities and travelers' characteristics, (2) expanding the amount of observed data (e.g., link counts and intersection counts), (3) calibrating parameters, and (4) enhancing the proposed approach such that the congestion impact analysis considers not only bicycle congestion, but also motorized congestion.

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