MOUNTAIN-PLAINS CONSORTIUM

MPC 15-283 | T. Chantem and D. Desiraju

Real-Time Traffic
Management to Maximize
Throughput of Automated
Vehicles

Photo: Ford

Real-Time Traffic Management to Maximize Throughput of Automated Vehicles

Tam Chantem, Ph.D
Divya Desiraju

Utah Transportation Center
Utah State University
Logan, Utah

March 2015

Acknowledgements

Disclaimer

ABSTRACT

In intelligent transportation systems, most of the research work has focused on lane change assistant systems. No existing work considers minimizing the disruption of traffic flow by maximizing the number of lane changes while eliminating the collisions. In this thesis, we develop qualitative and quantitative approaches for minimizing the disruption of traffic flow for three lane scenarios and show that we can extend our approach to an arbitrary number of lanes. The proposed algorithm is able to achieve the maximum number of lane changes. Simulation results show that our approach provides much better performance when compared with different lane change algorithms without incurring large overhead, and is hence suitable for online use.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

1. INTRODUCTION

Traffic congestion has become a major challenge for transportation professionals and roadway users across the world. As more of the world becomes more mobile, congestion during peak hours results in wasted time for billions of people around the globe. The effects of congestion delays on the individual are mostly negative: a reduction of air quality due to vehicle idling, drivers' quality of life affected by having large amounts of non-productive time, which results in reduced time with family and friends, as well as economic losses due to non-productivity. Congestion also has a negative impact on safety, as it causes drivers to make increased decisions during stop and go traffic.

Financial, environmental, and land use considerations provide an increasingly difficult environment to significantly increase the capacity of roadways by adding additional lanes. Fortunately, congestion can be alleviated by replacing human-operated vehicles with automated vehicles, which free the driver from the mental workload of a large number of tasks, some of which have to be carried out in parallel [1]. The promise of reduced non-recurring congestion, due to reduction in vehicle crashes (approximately 25% of all congestion in the U.S.), provides great opportunities for the supplement of automated vehicles into the fleet. In addition, computer-operated vehicles have shorter reaction times [2], which allow the vehicles to be closer to one another, thus increasing traffic flow.

Of all basic vehicular maneuvers, lane changing is arguably one of the most difficult ones. There were approximately 539,000 two-vehicle lane change crashes United States alone in 1999 [3]. Analysis of the German In-Depth Accident Study [3] from 1985 to 1999 shows that, on average, more than 5% of accidents occurred while changing lanes. In 2008, 1.7% of the registered highway accidents in the Netherlands were caused by inadequate lane changing [4]. While it has been shown by Tsao et al. that the exit success percentage, which is the number of automated vehicles that successfully exit the system divided by the number of vehicles that need to exit, is well below 100% due to the lack of gaps sufficiently large enough for safe lane changes [5]. We believe that it is crucial to provide a mechanism that best utilizes available gaps. To achieve the promise of high throughput and increased safety, a technique that minimizes the disruption of traffic flow by automated vehicles during lane changes must be implemented. In this article, we are interested in designing such a technique with the objective of maximizing the safe number of possible lane changes. Although there exists a large number of automated lane change assistant systems, as shown in Section II, to the best of our knowledge, there has been no work that attempts to minimize the disruption of traffic flow by maximizing the number of lane changes for live traffic on a stretch of a highway with an arbitrary number of lanes, without any assumptions on vehicles' dynamic attributes such as speeds.

Our main contributions are as follows.

- Given an arbitrary number of automated vehicles, we design an algorithm to maximize the number of possible lane changes on an arbitrary segment of a highway at any given time. The proposed algorithm uses information such as vehicles' positions, speeds, and time slacks (to be defined later) to make judicious lane change decisions without requiring prior knowledge on traffic patterns or unnecessary braking. To reduce runtime overhead, we propose a distributed approach to allow for local lane changing decisions to be made during run time.
- We present a lane change simulation platform that enables the implementation and comparison of different lane change algorithms. A large number of simulations can be run efficiently and various simulation parameters such as the number of vehicles wanting to change lanes can be specified.

The remainder of the thesis is outlined as follows. We review existing literature regarding lane changes in Section 2. Section 2 also provides the system model and state the assumptions made in the thesis. The minimum time slack calculations, which is used to determine if a vehicle can change lanes without a collision, is presented in Section 3. Our distributed approach is discussed in Section 4 and the details of our online algorithm in Section 5. Section 5 also discusses the practical factors involved in implementing our approach in real operating scenarios. Simulation results presented in Section 6 and Section 7 conclude the thesis.

2. RELATED WORK

Lane changing is an important topic that has received significant research attention. Some work focuses on lane change assistant systems for human drivers [4, 6, 7], while others consider lane change collision avoidance systems [8–12]. Chee and Tomizuka studied the lane change maneuver that is most comfortable to passengers [13,14]. The overtaking maneuver, which consists of one lane change from the right lane to the left lane and one lane change from the left to the right lane to pass a vehicle, has also been examined [15, 16].

Research that has been conducted yielded several sophisticated lane change controller designs [17–19]. A technique to perform lane changing to avoid obstacles is presented by Papadimitriou and Tomizuka [20]. A large number of automated lane change maneuver systems have been proposed to assist human drivers [21–26]. In particular, a neural network [24] and Bayesian data fusion [25] approaches are used to enable safe lane changes. Jula et al. presented some analysis to determine the minimum longitudinal spacing needed by a lane change [26].

In addition, to increase passenger safety, several researchers have presented various models to predict a vehicle's lane changing intention. For example, Xuan and Coifman exploited the availability of differential GPS data to detect lane changes [27]. Angkititrakul et al. used a stochastic driver behavior to predict whether a lane change may occur [28]. Many cooperative approaches that make use of vehicles-to-vehicles (V2V) communications exist for a variety of lane change related purposes: eliminating risks during lane change [29], merging due to lane closures [30] and freeway entrance [31], overtaking assistance [32], and path predictions for increased safety [33]. Ardelt et al. proposed a probabilistic framework for automated vehicles on freeways [34].

Despite the wealth of research on lane changes of automated vehicles, most work assume a two-lane (in either direction) system, consider only one lane change at any given time, or assume that the vehicles travel at about the same speed [35–37]. Hilscher et al. presented a method to perform lane change safety verifications of an arbitrary number of automated vehicles on a multi-lane highway [38], but do not attempt to maximize the number of lane changes at a given time.

A closely related topic to lane changing is the lane assignment problem where automated vehicles are assigned to their given lanes for each segment of the highway. Hall and Caliskan formulated this scheduling problem as a linear optimization problem with the objective of throughput maximization [39, 40], while Ramaswamy et al. opted to minimize travel time [41]. Lane assignments may be made for each vehicle or a group of vehicles and usually depend on the distance a vehicle has to travel before exiting the highway. The lane assignment problem can also be solved using genetic algorithms, as shown by Kim et al. [42]. The key assumptions made in previous work is that (i) the automated vehicles travel at about the same speed for a given lane or across all the lanes, and/or that this speed does not depend on traffic volume, and (ii) traffic is known a priori. In contrast, we do not rely on either of these assumptions in this work.

2.1 System Model and Assumptions

We consider a set of automated vehicles Ψ along an arbitrary segment of an m-lane highway, where m is an integer and $m \geq 2$. The width W of each lane is known *a priori*. Although we assume for the sake of simplicity that all lane widths are equal, this work can readily be applied to highways in which lane widths differ. Each automated vehicle V_i is characterized by its length l_i and width w_i. At any given time, the current lane, velocity u_i, acceleration a_i, and jerk j_i are known. In addition, the position p_i of the front left of the vehicle with respect to some reference point, which is represented by a tuple (x_i, y_i), is known for vehicle V_i. Figure 2.1 shows a six-lane highway example with three automated vehicles. At any point

in time, a vehicle may wish to perform a lane change for whatever reason. For instance, a vehicle V_i may want to change lanes since it is coming upon a slower moving vehicle V_j in front of it. In such a case, if a lane change is not made (or not made until later), V_i will slow down and adopt the Gipps' car following model [43], which is a widely used car following model. That said, our approach can be modified for use with other car following models.
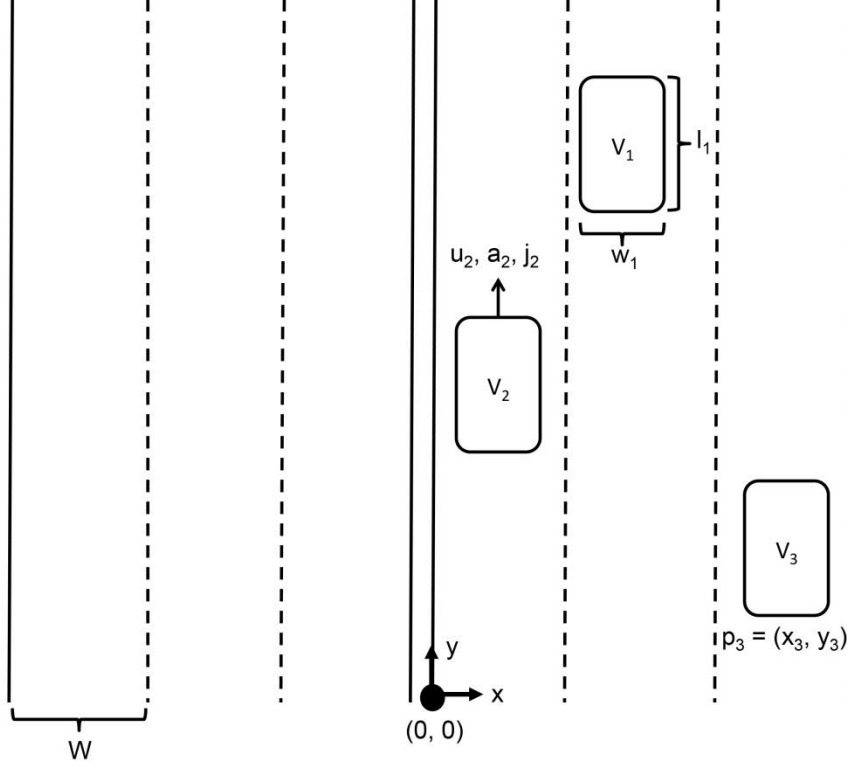
Figure 2.1 An example six-lane highway.

We assume the existence of either a roadside infrastructure, which allows for vehicle-to-infrastructure (V2I) communications [44, 45], or a vehicular ad hoc network (VANET) for vehicle-to-vehicle (V2V) communications [33]. Such communications are used by a vehicle to obtain necessary information (e.g., velocity, acceleration, etc.) of other vehicles in the vicinity.

The distance traveled by a vehicle V_i during the time interval $[t_0, t]$ is

$$s_i(t) = s_i(t_0) + u_i(t - t_0) + \frac{1}{2}a_i(t - t_0)^2 + \frac{1}{6}j_i(t - t_0)^3 \qquad (2.1)$$

In this article, we adopt the approach used by Neades and Ward [46] to compute the time a vehicle V_i requires to perform a lane change. Specifically, the objective of the original analysis is to compute the minimum time taken to change lanes given the critical speed, which is the maximum speed at which a turn can be negotiated [46]. Significant modifications were made to the original derivation to obtain the time required to perform a lane change for a given vehicle with arbitrary velocity, acceleration, and jerk. That is, the swerve taken by vehicle V_i follows the trajectory (dotted line) illustrated in Figure 2.2. Here, a is assumed to be half the width of a lane and thus is known. The angle θ_i is also known since we are considering automated vehicles.

4

With known value of θ_i and a_i, $c_i = \frac{a_i}{\cos \theta_i}$. Applying Pythagorean theorum, $b_i = \sqrt{(c_i^2 - a_i^2)}$. The total distance vehicle V_i requires to perform a lane change (i.e., complete swerve) is $d_i = 2 \left(\frac{\pi}{2} b_i\right) = \pi b_i$. Finally, the tiem to lane change t_i^c for vehicle V_i can be found by solving the following equation

$$\pi b_i = u_i t_i^c + \frac{1}{2} a_i (t_i^c)^2 + \frac{1}{6} j_i (t_i^c)^3. \tag{2.2}$$

For the sake of clarity, we ignore lateral acceleration. However, said acceleration can be incorporated when calculating the time to lane change. The proposed technique requires no modification when lateral acceleration is considered.
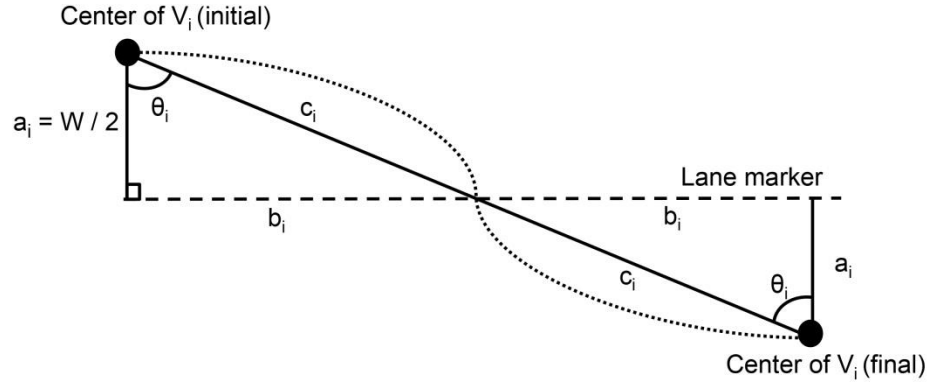

Figure 2.2 Lane change maneuver of an automated vehicle [46].

3.   MINIMUM TIME SLACK CALCULATIONS

Let us consider an automated vehicle V_i whose attributes are as described in Chapter 2, System Model and Assumptions. As shown in that section, the time to lane change for V_i can be computed as in Equation 2.2 and depends on a number of factors such as V_i's speed, as well as the lane width. However, since V_i is unlikely to be the only vehicle on a given stretch of highway, V_i may not be able to change lanes right away or a collision may ensue if the gap between V_i and another vehicle is not large enough. We now use a simple example to demonstrate how the time vehicle V_i *has* to change lanes can be calculated.
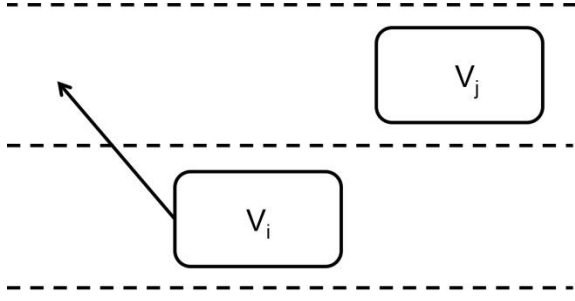
Figure 3.1 An example scenario where V_i wishes to change into V_j's lane.

Figure 3.1 shows an example scenario consisting of two automated vehicles V_i and V_j at some time t. Let the current positions of V_i and V_j be $p_i = (x_i, y_i)$ and $p_j = (x_j, y_j)$, respectively. In addition, V_i is in front of V_j, i.e., $y_i \geq y_j$. Let us assume V_i starts the lane change process at time t and both vehicles maintain their velocities, accelerations, and jerks. Let $\acute{p}_\iota = (\acute{x}_\iota, \acute{y}_\iota)$ be the new position of V_i at time $t + t_i^c$. In addition, let V_j's position at time $t + t_i^c$ be $\acute{p}_J = (\acute{x}_J, \acute{y}_J)$. Clearly, $\acute{x}_J = x_j = \acute{x}_\iota$, and

$$\acute{y}_\iota = y_j + u_j(t_i^c) + \frac{1}{2}a_j(t_i^c)^2 + \frac{1}{6}j_j(t_i^c)^3. \tag{3.1}$$

A collision will not occur if, at time $t + t_i^c$, V_i either remains behind V_i and the latter's headway is at least three seconds or V_j is now in front of V_i and its headway is at least three seconds. For the first scenario to be true, the following must be satisfied

$$\acute{y}_\iota - l_i \geq r(v_i, a_i, j_j) + \acute{y}_\iota. \tag{3.2}$$

where l_i is the length of V_i and $r(v_i, a_i, j_j)$ is the minimum distance between V_i and V_j according to the three-second following distance rule, which depends on v_i, a_i, and j_j. Similarly, if V_j is now in front of V_i, we have

$$\acute{y}_J - l_j \geq r(v_i, a_i, j_j) + \acute{y}_\iota. \tag{3.3}$$

Consequently, $t_{i,j}^h$, the time V_i has to change lane with respect to V_j, can be obtained by solving the following expression

$$\acute{y}_\iota - l_i - r(v_i, a_i, j_j) = u_j(t_{i,j}^h) + \frac{1}{2}a_j(t_{i,j}^h)^2 + \frac{1}{6}j_j(t_{i,j}^h)^3, \tag{3.4}$$

provided that V_i will end up in front of V_j. A similar condition can be derived for the case where V_j will be in front of V_i.

We are ready now to define the time slack of V_i with respect to V_j.

Definition 1. *The time slack of V_i will end up in front of V_j is the difference between the time V_i has to change lanes with respect to V_j and the time V_i takes to change lanes given its current velocity, acceleration, and jerk. In other words*,

$$sl_{i,j} = t_{i,j}^h - t_c^i. \qquad (3.5)$$

The time slack helps to determine whether a lane change is safe. That is, a positive time slack denotes a safe lane change (with respect to another vehicle) while a negative time slack implies that a collision may occur. In real scenarios, a vehicle wanting to change lanes may need to consider its time slacks with respect to a number of vehicles, instead of just one vehicle.
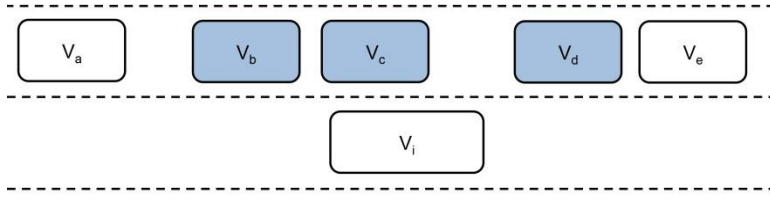
Figure 3.2 If V_i wishes to change into the top lane, it must consider its time slacks with respect to the shaded vehicles.

Figure 3.2 indicates the vehicles that V_i (the vehicle wanting to change lanes) needs to account for. Let Γ be the set of vehicles currently in the lane that V_i wishes to change to. Then, the time slack of V_i with respect to $V_j \in \Gamma$ needs to be computed if

- V_j laterally overlaps with V_i, i.e., $y_i - l_i \le y_j \le y_i$ or $y_i - l_i \le y_j - l_j \le y_i$,
- V_j is the lateral vehicle immediately in front of V_i, i.e., $y_j = min_{V_k \in \Gamma}\{y_k\} | y_j > y_i$ and V_j is not traveling faster than V_i, or
- V_j is the lateral vehicle immediately behind V_i, i.e., $y_j = max_{V_k \in \Gamma}\{y_k\} | y_j < y_i - l_i$.

We are now ready to generalize the concept of time slack.

Definition 2. *The minimum time slack of V_i with respect to a group of vehicles $\hat{\Gamma}$ is the minimum difference between the time V_i has to change lanes with respect to $V_j \in \Gamma'$ and the time V_i takes to change lanes given its current velocity, acceleration, and jerk. In other words*,

$$sl_i^* = \min_{V_j \in \Gamma'} sl_{i,j}. \qquad (3.6)$$

If at most, one vehicle wants to change lanes, a positive minimum time slack indicates that a safe lane change can take place. We next consider the more realistic scenarios where more than one vehicle on a segment of a highway may wish to change lanes.

4. A DISTRIBUTED APPROACH FOR LARGE HIGHWAYS

One way to maximize the number of lane changes given a set of automated vehicles on a stretch of highway is to formulate the problem as an optimization problem with constraints on safety for each time instant. However, the resultant optimization problem is relatively complex and contains integer variables, making it hard to solve the problem efficiently online using a mixed-integer programming solver. An alternative approach is to consider, for each stretch of the highway of interest, all the vehicles in all the lanes in order to make centralized, globally optimal decisions. However, this approach may not be practical or efficient enough when there is a large number of vehicles. In addition, such a centralized approach requires that each vehicle be aware of all other vehicles on that particular stretch of highway, even if they are far enough apart that they cannot possibly interfere with one another. For these reasons, we resort to designing efficient local algorithms. The key idea is to solve the problem in a distributed manner instead of globally.

We observe that given an m-lane highway in each direction, we can divide the problem of lane change maximization into a number of sub-problems, as illustrated in Figure 4.1. In this example, there are five lanes and 16 vehicles, eight of which want to change lanes. To reduce runtime overhead, a sub-problem is created for each lane that at least one vehicle wants to change to. There are four sub-problems in this example, as no vehicle wishes to change to the top lane. In sub-problem 1 (Figure 4.1b), potential changes into the second lane from the top are considered. For this reason, potential lane changes by V_{21} and V_{23} are ignored since these vehicles may or may not change lanes in the end. This process is repeated for all the lanes. Algorithm 1 provides the steps needed to create the sub-problems. It takes as inputs the number of lanes and the set of vehicles, and returns a set of sub-problems. Each sub-problem consists of a number of lanes, the vehicles in each of the lanes, and a set of vehicles that wish to change into a common lane.
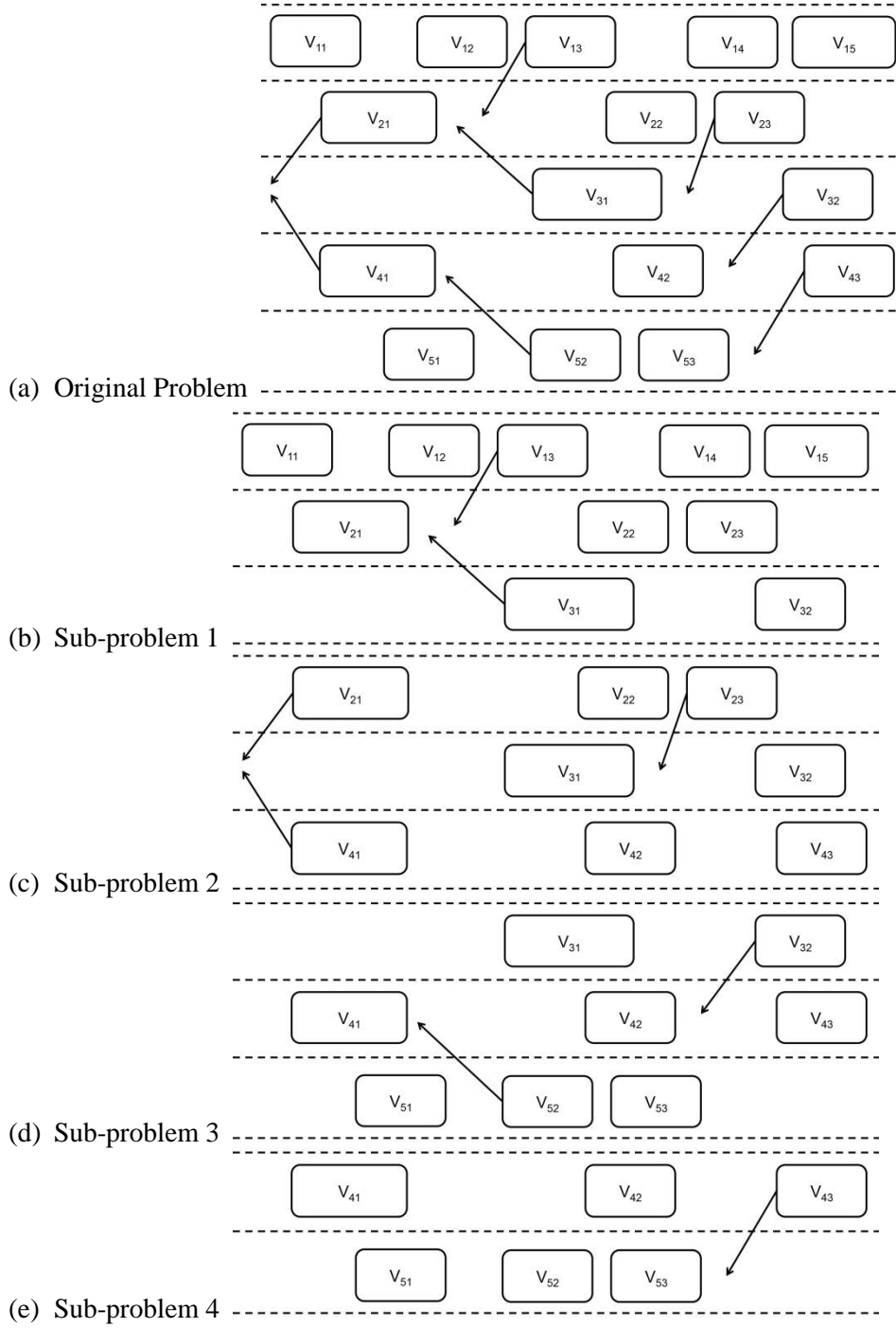
Figure 4.1 An example to illustrate how the lane change maximization problem on a five-lane highway in each direction can be considered four lane change maximization problems on three three-lane highways and one two-lane highway. The arrow in front of a vehicle indicates the vehicle's desire to perform a lane change. In Sub-problem 1, only changes into the second lane are considered. This is the reason why the potential lane change by V_{21} and V_{23} are not considered in this sub-problem.

Algorithm 1: Divide_Into_Subproblems(m, Ψ)

1: $\Psi_i \leftarrow \emptyset, i = 1, \dots, m$

2: **for** $i - 1, \dots, m$ **do**

3:     **for** each $V_j \epsilon \Psi$ **do**

4:         **if** $V_j currLane = i$ **then** {$V_j's$ current lane is L_i}

5:             $\Psi_i \leftarrow \Psi_i \cup V_j$

6:         **end if**

7:     **end for**

8: **end for**

9: **for** $i = 1, \dots, m$ **do**

10:     **if** $i = 1$ **then**

11:         **if** $\exists V_j \in \Psi_{i+1} | V_j.desiredLane = i$ **then**

12:             Create_Sub-problems($2, L_i, L_{i+1}, \Psi_i, \Psi_{i+1}$)

                {C}reate a sub-problem with 2 lanes L_i and L_{i+1} containing all the vehicles in Ψ_i and Ψ_{i+1}

13:         **end if**

14:     **else if** $i = m$ **then**

15:         **if** $\exists V_j \in \Psi_{i-1} | V_j.desiredLane = i$ **then**

16:             Create_Sub-problems($2, L_{i-1}, L_i, \Psi_{i-1}, \Psi_i$)

                {C}reate a sub-problem with 2 lanes L_{i-1} and L_i containing all the vehicles in Ψ_{i-1} and Ψ_i

17:         **end if**

18:     **else**

19:         **if** $\exists V_j \in \Psi_{i-1} \cup \Psi_{i+1} | V_j.desiredLane = i$ **then**

20:             Create_Sub-problems($3, L_{i-1}, L_i, L_{i+1}, \Psi_{i-1}, \Psi_i \Psi_{i+1}$)

                {C}reate a sub-problem with 3 lanes L_{i-1}, L_i, and L_{i+1} containing all the vehicles in Ψ_{i-1}, Ψ_i, and Ψ_{i+1}

21:         **end if**

22:     **end if**

23: **end for**

The time complexity of Algorithm 1 is O(m · |Ψ|), where *m* is the number of lanes on the stretch of the highway under consideration and |Ψ| is the number of vehicles associated with said stretch of the highway. To prove some properties of the sub-problems created using Algorithm 1, we start with a definition followed by a lemma.

Definition 3. *A feasible lane change configuration within a sub-problem is a set of lane change decisions made within that sub-problem that ensures no collision among vehicles within the sub-problem will occur.*

Lemma 1. *Consider an m-lane highway in each direction, a set of automated vehicles Ψ, and a set of automated vehicles wanting to change lane Λ where Λ ⊆ Ψ. Applying Algorithm 1 will result in at most m sub-problems. In addition, decisions whether or not to allow vehicles in each sub-problem to change lanes can be made independently, i.e., without considering decisions made in other sub-problems, and no collision will occur due to these independent lane change decisions as long as the lane change configuration within each sub-problem is feasible.*

Proof: It is straightforward to show that there can be at most *m* sub-problems, since there can be at most one sub-problem per lane. We now show that no collision can occur by making lane change decisions for each sub-problem in parallel.

Without loss of generality, let us assume that there are two sub-problems S_1 and S_2 for changes into lanes L_1 and L_2, respectively. In addition, a feasible lane change configuration within each sub-problem is found, i.e., there are no collisions among vehicles within the sub-problem. Now, let us assume that applying said feasible lane change configurations result in a collision. Since, by definition, a feasible lane change configuration ensures no collision among vehicles within a sub-problem can happen, a collision must occur outside of the sub-problems, i.e., in the original problem. Since sub-problem S1 focuses on changes into lane L_1 and sub-problem S_2 lane L_2, a collision can only occur if a vehicle from lane L_1 does not safely change into lane L_2 (or vice versa). However, during the creation of the sub-problems, all the vehicles in a given lane are considered. Hence, a collision cannot happen. This is a contradiction and the lemma is proved.

Based on the above lemma, we will now focus on the problem of maximizing the number of lane changes on a three-lane highway.

5.   SIMULATIONS

We are interested in solving the following problem.

Problem 1: Given a three-lane highway with a set of automated vehicles whose attributes such as velocity, acceleration, and jerk are known, and in which a subset of those vehicles wish to change lanes, determine the set of vehicles that are allowed to change lanes in order to maximize the total number of lane changes at any given time.

Although it has been shown in the previous section that an m-lane highway can be divided into several three-lane highways to reduce the complexity of the problem, the number of automated vehicles on a given stretch of a highway may still be large. To further optimize for the efficiency of our approach, we now introduce the concept of *grouping* of vehicles, which will allow us to solve Problem 1 in a distributed manner.

The main idea behind grouping is based on the observation that several lane changes may occur at the same time on a given stretch of a three-lane highway, as long as vehicles are far enough apart, as shown in Figure 5.1a. This idea can be taken a step further, as illustrated in Figure 5.1b, by observing that grouping can be made with respect to some vehicle. For example, in Figure 5.1b, V_A can change lanes without needing to consider V_D, but must account for both V_B and V_C, as the latter vehicles are within its "range." Our concept of grouping allows for only for a small group of vehicles to be considered for simultaneous Type equation here.lane change decisions. We now present our grouping algorithm, which is shown in Algorithm 2.

(a) If V_A and V_B are far enough apart from the rest of the vehicles, they can be considered separately from the other vehicles when making lane change decisions.
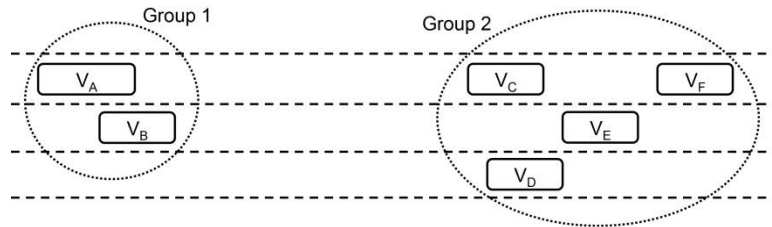
(b) Here V_A must be considered V_B and V_C but can ignore V_D.
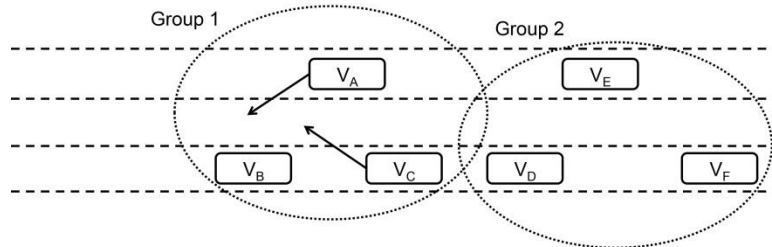
Figure 5.1 Grouping examples

Algorithm 2 takes as input Ψ, which the[MS1] set of vehicles on a three-lane highway. The first step taken by Algorithm 2 is to sort the vehicles such that $\forall V_i, V_j \in \Psi, i < j$ if and only if $y_i < y_j$. In other words, vehicles are sorted in a non-increasing order of their y positions. Algorithm 2 then starts a group containing V_i, which is the first vehicle in Ψ. Next, using V_i's time to change lane t_c^i, it computes the distance separating V_i and V_j (the next vehicle in Ψ), including the three-second following distance rule. If this distance d_s is negative, a collision may occur if V_i and V_j change lanes at the same time. As a result, V_j must be grouped with V_i and Algorithm 2 continues the same process with the next vehicle in Ψ. Otherwise, the current grouping is finished and the new group is started until there are no vehicles remaining in Ψ. An optimization can be made to Algorithm 2 by only including vehicles that wish to change to the common lane and the vehicles already in that lane. This is because vehicles that do not currently wish to change lanes and that are not currently in the common lane cannot interfere with those wishing to switch lanes.

Algorithm 2: Vehicles_Grouping (Ψ)

1:    $\Psi \leftarrow \Psi$ sorted in a non-increasing order of $y_i, i = 1, \dots, |\Psi|$ {S}ort vehicles by their positions, with the first vehicle being the one in front of other vehicles. Ties broken in favor of smaller time slacks.

2:    $\Upsilon \leftarrow \emptyset$ {Υ will hold the final groupings}

3:    **while** $\Psi \neq \emptyset$ **do**

4:        $V_i \leftarrow \Psi[0]$ {V_i is the first vehicle in Ψ}

5:        $\Psi \leftarrow \Psi - V_i$ {Remove V_i from the set of vehicles}

6:        $v \leftarrow V_i$ {The current grouping contains V_i}

7:        **while true do**

8:           $V_j \leftarrow \Psi[0]$

9:           $\acute{y_i}' = u_i(t_c^i) + \frac{1}{2} a_i(t_c^i)^2 + \frac{1}{6} j_i(t_c^i)^3$

10:         $\acute{y_j} = \left(u_j(t_c^i) \frac{1}{2} a_j(t_c^i)^2 + \frac{1}{6} j_j(t_c^i)^3\right)$

11:         **if** $\acute{y_i} > \acute{y_j}$ **then** {V_i will be in front of V_j}

12:             $d_s \leftarrow \acute{y_i} - l_i - \acute{y_j} - r(v_i, a_i, j_i)$

13:         **else** {V_j will be in front of V_i}

14:             $d_s \leftarrow \acute{y_j} - l_j - \acute{y_i} - r(v_j, a_j, j_j)$

15:         **end if**

16:         **if** $d_s < 0$ **then** {If a collision will occur}

17:             $v \leftarrow v \cup V_j$ {Include V_j inside this group since V_j can interfere with V_i}

18:             $\Psi \leftarrow \Psi - V_j$ {Remove V_j from the set since V_j has already been grouped}

19:         **else** {Need to start a new group}

20:             $\Upsilon \leftarrow v$

21:             **break** {Go back to Line 4}

22:         **end if**

23:        **end while**

24: **end while**

25: **return** Υ

The time complexity of Algorithm 2 is $O(|\Psi|^2)$, since sorting takes $O(|\Psi| \cdot \log|\Psi|)$ and the most time consuming part of the algorithm occurs within the while loop. In the worst case, one vehicle is removed from Ψ in every iteration, which means that the while loop will iterate for at most $|\Psi|$ times. In addition, the inside while loop will iterate for at most $|\Psi|$ times, while all other operations take constant time. The time complexity of Algorithm 2 can be reduced to $O(|\Psi| \cdot \log|\Psi|)$ by replacing the inner while loop with a for loop and using binary search.

It is worth noting that some checkpoints are left off the description of Algorithm 2 for the sake of clarity. For example, additional steps are needed if there exist at least two vehicles with exactly the same y values, i.e., $\exists y_i = y_j$, V_i, $V_j \in \Psi$.

Once grouping takes place, the vehicle at the front of each group will be selected for lane change. We now discuss some properties of Algorithm 2 using the following lemmas and theorem.

Lemma 2. *Consider a three-lane highway with a set of automated vehicles Ψ. If Algorithm 2 is used to group vehicles in such a way that one vehicle per group performs a lane change, no collisions will take place.*

Proof: The proof is straightforward, as a new group is formed by Algorithm 2 if the safety distance computed on Line 9 is satisfied.

Lemma 3. *Consider a three-lane highway with a set of automated vehicles Ψ, applying Algorithm 2 results in the maximum number of groups where one vehicle per group can change lanes without violating safety constraints.*

Proof: We prove the lemma using contradiction. Let us suppose that Algorithm 2 found n groups, but that a feasible solution with $n + 1$ groups exists. Without loss of generality, let us also assume that in the second, i.e., better, set of solutions, the vehicles in the nth and $n + 1^{\text{th}}$ groups make up the nth group found by Algorithm 2. This means that it is possible to divide the nth group found by Algorithm 2 into two (or more) groups. However, in Algorithm 2, a new group is formed only if the safety constraint (Line 9) is satisfied. This violates the original assumption that the second set of solution is feasible. Hence, the lemma is proved.

Theorem 1. *Consider a three-lane highway with a set of automated vehicles Ψ, some of which wish to switch to the center lane. Using Algorithm 2 to group the vehicles and selecting the vehicle at the front of each group for lane change results in the maximum number of lane changes, provided that only one vehicle per group is allowed to change lanes at a given time instant.*

Proof: The proof directly follows from Lemmas 2 and 3.

5.1   Practical Considerations

Algorithms 1 and 2 were described in such a way as to facilitate the discussions. The use of Algorithm 1 in real operating scenarios is straightforward; the "center" lane is always the lane vehicles wish to change to. Hence, for an *m*-lane highway (in each direction), there can logically be up to six "center" lanes.

As for Algorithm 2, information regarding groups must be passed downstream, i.e., from vehicles in the front to the ones in the back. However, the process can be optimized whenever situations similar to the one in Figure 5.1a arise. That is, since V_C can obtain information regarding the position, velocity, acceleration, and jerk of V_B, V_C can easily determine if it can form its own group that is separate from V_B.

6. SIMULATIONS

We compare the effectiveness and efficiency of our proposed algorithm against the following techniques, both analytically and using simulations. Note that comparison choices are very limited, as we are the first to consider the problem of lane change maximization. To ensure fair comparison, an m-lane highway (in each direction) is divided into several three-lane highways as discussed in Chapter 4.

- Random algorithm: A number between [*0, k*] is randomly generated, where k is the number of vehicles that wish to make a lane change. Based on this random number *r, r* vehicles will randomly be selected for lane change.
- Greedy algorithm: In this algorithm, the minimum time slacks are ignored and all the vehicles that want to change lanes will be allowed to change lanes.
- Least slack first algorithm: One vehicle is selected to change lanes at any point in time. The vehicle with the minimum time slack will be chosen.

6.1 Simulation Framework

Since the objective of the simulations is to evaluate the performance of the proposed algorithm compared to the baseline algorithms, we assume that information on surrounding vehicles, such as positions, velocities, and accelerations, are readily available. (The information would in reality be sent to the vehicles using either V2V or V2I.) Specifically, for each vehicle in a given time instant, the following values are known to the system: unique vehicle ID, position, velocity, acceleration, jerk, safe distance, with respect to the vehicle immediately in front of it, according to the three-second rule, θ (the angle at which the vehicle takes to perform a lane change, see *System Model and Assumptions* in Chapter 3), time taken to perform a lane change, current lane, and desired lane. If a vehicle does not wish to change lanes at this time, then the current lane is the same as the desired lane.

We randomly generated 20,000 benchmarks, each of which contains a number of automated vehicles on a three-lane highway in each direction. The highway is assumed to have three lanes since we have previously shown that the problem of lane change maximization on wider highways can be divided into a number of sub-problems with three-lane highways. The total number of vehicles in a benchmark ranges from 5 to 100, with the number of vehicles wishing to change lanes being between 0 and 55. For the sake of simplicity, all vehicles are assumed to have the same width, length, and θ, and jerks are set to zero. The positions, velocities, accelerations, as well as starting and end lanes were randomly generated. The ranges for these values can be found in Table 6.1. Given these values, the safety distance (the minimum distance separating this vehicle from the vehicle directly in front of it) and the time the vehicle takes to change lane, can be computed.

Table 6.1 The ranges use for the various attributes of the vehicles used in the simulations

Vehicle Attribute	Minimum Value	Maximum Value
Y-Position	0	1600
Velocity (*m/s*)	5	30
Acceleration (*m/s^2*)	0	2

The following performance metrics will be used in each benchmark to assess the performance of our algorithm: lane change ratios, collision ratios, and time overheads. The lane change ratio l is defined as

$$l = \frac{Number\ of\ safe\ lane\ changes\ performed}{Total\ number\ of\ desired\ lane\ changes}, \tag{6.1}$$

while the collision ratio c can be expressed as

$$c = \frac{Number\ of\ collisions}{Total\ number\ of\ vehicles}. \tag{6.2}$$

Finally, the time overheads represent the overheads associated with all the algorithms and will indicate whether our proposed method is suitable for online use.

6.2   Analytical Comparisons

Before presenting the simulation results, we analytically derive the best- and worst-case scenarios for the algorithms. As will be shown in the next section, the simulation results verify the analyses presented here.

Let k and n be the number of vehicles that wish to change lanes and the number of groups when using the proposed algorithm, respectively. The best and worst cases are shown in Tables 6.2 and 6.3, respectively. Thanks to our grouping method, no collisions will occur. The proposed algorithm results in the maximum number of lane changes, provided that, at most, one vehicle per group can change lanes. For both the random and greedy algorithms, the worst case occurs when every lane change results in a collision (r is the random number generated by the random algorithm and represents the number of vehicles allowed to change lanes using that algorithm). In contrast, the least-slack first algorithm ensures that exactly one safe lane change is performed at any point in time.

Table 6.2 Worst case performance of different algorithms

Algorithm	Number of Collisions		Number of Safe Lane Changes
Proposed	0		$n, 1 \leq n \geq k$
Random	r		0
Greedy	k		0
Least-Slack First	0		1

The best-case scenarios for the proposed algorithm and the least-slack first algorithm are the same as in the worst-case scenarios. In the best case, using the random and greedy algorithms will result in no collisions. Clearly, our proposed technique never performs worse than the other algorithms and has a much better performance in the worst-case scenario.

Table 6.3 Best case performance of different algorithms

| Algorithm	Number of Collisions	Number of Safe Lane Changes
Proposed	0	k
Random	0	r
Greedy	0	k
Least-Slack First	0	1

6.3   Simulation Results

The average lane change ratio for the different algorithms is shown in Figure 6.1a. It is clear from the plots that our proposed algorithm outperforms the baseline algorithms by significant margins. The maximum, minimum, and average percent improvements in lane change ratio of our method over the other algorithms are shown in Table 6.4. Figure 6.1b depicts the average collision ratio for the algorithms. Both our method and the least-slack first algorithm resulted in no collisions, while, as expected, the greedy algorithm has the highest collision ratio.

Table 6.4 Minimum, maximum, and average percent improvement of our proposed approach over the baseline algorithms in terms of lane change ratio.

% Improv. on Lane Change Ratio	Minimum	Maximum	Average
Greedy	42.3	108.7	67.6
Least-Slack First	50.5	2454.5	1385.8
Random	44.4	438.7	298.5

(a) Average lane change ratio as a function of number of vehicles for the different algorithms.
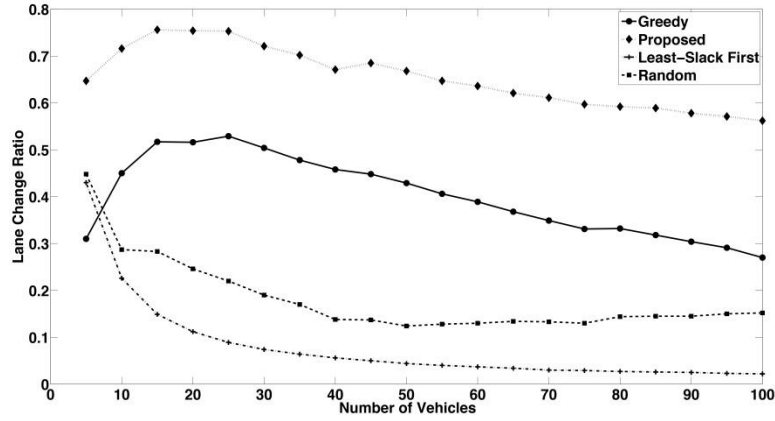
(b) Average collision ratio as a function of number of vehicles for the different algorithms.
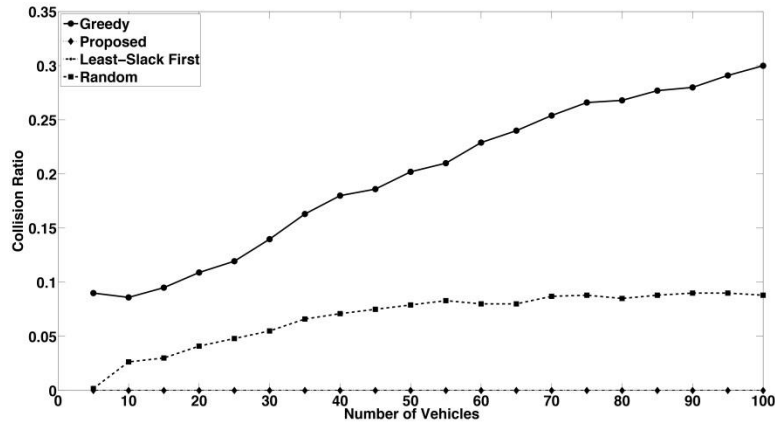
(c) Average time overhead in seconds as a function of number of vehicles for the different algorithms.
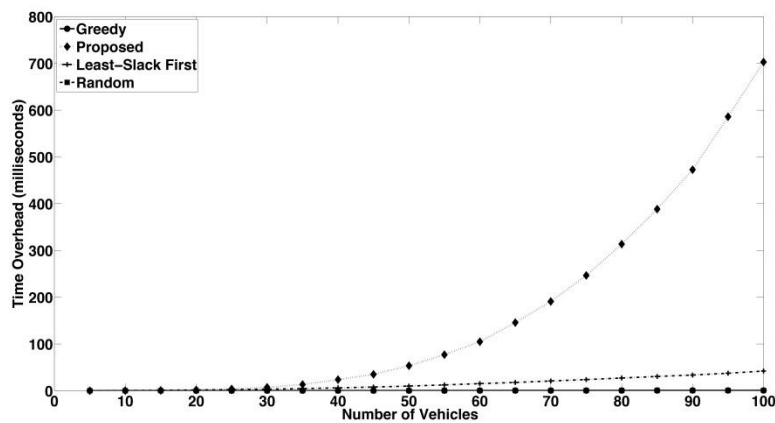

Figure 6.1 Simulation Data

From the previous data, it is clear that our proposed method achieves the best performance in terms of lane changes and collision avoidance. The average time overhead of our algorithm compared to the other methods is shown in Figure 6.1c based on the simulations conducted on an Intel i7 3.50GHz with 16GB memory. Since our algorithm is the most sophisticated, it is also the most time consuming approach.

To recap, the simulation data show that our proposed method can efficiently and effectively manage gaps between vehicles to allow for as many vehicles that need to change lanes to do so without causing collisions. We intend to improve the efficiency of our algorithm in future work. That said, the method presented in this thesis is appropriate for small- to mid-size lane change scenarios.

7. CONCLUSION

This report discussed the problem of lane change maximization of automated vehicles in order to minimize the disruption of traffic flow caused by lane changes. A distributed algorithm was proposed to solve the problem. The key ideas behind said algorithm are time slack calculations and the concept of vehicle grouping. Simulation results show that the proposed method increases the number of lane changes by up to 109%–2,454% and 68%–1,386% on average compared with a number of baseline algorithms.

This work can be extended in several directions. First, a mixed system consisting of automated and manual vehicles can be considered. Second, it would be useful to consider the urgency of a vehicle that wishes to change lanes in order to further minimize the disruption of traffic flow. For instance, a vehicle needing to take an exit should be given a higher priority. Third, accelerations during lane changes may be explored to further increase the number of lane changes. Finally, while it is helpful to maximize the number of lane changes to alleviate its disruptive effects on traffic flow, the problem of deciding whether an automated vehicle should change lanes in order to maximize throughput needs to be studied.

8. REFERENCES

[1] K. Ahmed, "Modeling drivers' acceleration and lane changing behavior," Ph.D. dissertation, Massachusetts Institute of Technology, 1999.

[2] T. Cowen, "Can I see your license, registration and C.P.U.?" 2011. [Online]. Available: http://www.nytimes.com/2011/05/29/business/economy/29view.html

[3] B. Sen, J. Smith, and W. Najm, "Analysis of lane change crashes," National Highway Traffic Safety Administration, Tech. Rep., 2003.

[4] M. Roelofsen, J. Bie, L. Jin, and B. V. Arem, "Assessment of safety levels and an innovative design for the lane change assistant," in Proceedings of the Intelligent Vehicles Symposium, Jun. 2010, pp. 83–88.

[5] H.S. Tsao, R. Hall, and B. Hongola, "Capacity of automated highway systems: effect of platooning and barrier," California Partners for Advanced Transit and Highways (PATH), Tech. Rep., 1994.

[6] S. Habenicht, H. Winner, S. Bone, F. Sasse, and P. Korzenietz, "A maneuver-based lane change assistance system," in Proceedings of the Intelligent Vehicles Symposium, Jun. 2011, pp. 375–380.

[7] G. Xu, L. Liu, Y. Ou, and Z. Song, "Dynamic modeling of driver control strategy of lane-change behavior and trajectory planning for collision prediction," IEEE Transactions on Intelligent Transportation Systems, vol. 13, no. 3, pp. 1138–1155, Sep. 2012.

[8] J. Bascunana, "Analysis of lane change crash avoidance," in Future Transportation Technology Conference and Exposition, Aug. 1995.

[9] S. Hetrick, "Examination of driver lane change behavior and the potential effectiveness of warning onset rules for lane change or side crash avoidance systems," master's thesis, Virginia Polytechnic Institute and State University, 1997.

[10] T. Wakasugi, "A study on warning timing for lane change decision aid systems based on driver's lane change maneuver," in Proceedings of the International Technical Conference on the Enhanced Safety of Vehicles, Jun. 2005.

[11] J. Li-sheng, F. Wen-ping, Z. Ying-nan, Y. Shuang-bin, and H. Hai-jing, "Research on safety lane change model of driver assistant system on highway," in Proceedings of the Intelligent Vehicles Symposium, Jun. 2009, pp. 1051–1056.

[12] A. Amditis, M. Bimpas, G. Thomaidis, M. Tsogas, M. Netto, S. Mammar, A. Beutner, N. Möhler, T. Wirthgen, S. Zipser, A. Etemad, M. Da Lio, and R. Cicilloni, "A situation-adaptive lane-keeping support system: Overview of the safelane approach," *IEEE Transactions on Intelligent Transportation Systems*, vol. 11, no. 3, pp. 617–629, Sep. 2010.

[13] W. Chee and M. Tomizuka, "Vehicle lane change maneuver in automated highway systems," California PATH Research Report, Tech. Rep., 1994.

[14] ——, "Unified lateral motion control of vehicles for lane change maneuvers in automated highway systems," California PATH Research Report, Tech. Rep., 1997.

[15] F. Wang, M. Yang, and R. Yang, "Conflict-probability-estimation-based overtaking for intelligent vehicles," *IEEE Transactions on Intelligent Transportation Systems*, vol. 10, no. 2, pp. 366–370, Jun. 2009.

[16] J. Naranjo, C. Gonzalez, R. Garcia, and T. de Pedro, "Lane-change fuzzy control in autonomous vehicles for the overtaking maneuver," *IEEE Transactions on Intelligent Transportation Systems*, vol. 9, no. 3, pp. 438–450, Sep. 2008.

[17] J. Lygeros, D. Godbole, and S. Sastry, "Verified hybrid controllers for automated vehicles," *IEEE Transactions on Automatic Control*, vol. 43, no. 4, pp. 522–539, Apr. 1998.

[18] R. Rajamani, H.-S. Tan, B. Law, and W.-B. Zhang, "Demonstration of integrated longitudinal and lateral control for the operation of automated vehicles in platoons," *IEEE Transactions on Control Systems Technology*, vol. 8, no. 4, pp. 695–708, Jul. 2000.

[19] C. Hatipoglu, U. Ozguner, and K. Redmill, "Automated lane change controller design," *IEEE Transactions on Intelligent Transportation Systems*, vol. 4, no. 1, pp. 13–22, Mar. 2003.

[20] I. Papadimitriou and M. Tomizuka, "Fast lane changing computations using polynomials," in Proceedings of the American Control Conference, Jun. 2003, pp. 48–53.

[21] L. Wan, P. Raksincharoensak, and M. Nagai, "Study on automatic driving system for highway lane change maneuver using driving simulator," *Journal of Mechanical Systems for Transportation and Logistics*, vol. 4, no. 2, pp. 65–78, 2011.

[22] R. Schubert, K. Schulze, and G. Wanielik, "Situation assessment for automatic lane-change maneuvers," *IEEE Transactions on Intelligent Transportation Systems*, vol. 11, no. 3, pp. 607–616, Sep. 2010.

[23] W. He, X. Wang, G. Chen, M. Guo, T. Zhang, P. Han, and R. Zhang, "Monocular based lane-change on scaled-down autonomous vehicles," in Proceedings of the Intelligent Vehicles Symposium, Jun. 2011, pp. 144–149.

[24] R. Tomar and S. Verma, "Safety of lane change maneuver through a priori prediction of trajectory using neural networks," *Network Protocols and Algorithms*, vol. 4, no. 1, May 2012.

[25] R. Schubert and G. Wanielik, "Empirical evaluation of a unified bayesian object and situation assessment approach for lane change assistance," in Proceedings of the International Conference on Intelligent Transportation System, Oct. 2011, pp. 1471–1476.

[26] H. Jula, E. Kosmatopoulos, and P. Ioannou, "Collision avoidance analysis for lane changing and merging," *IEEE Transactions on Vehicular Technology*, vol. 49, no. 6, pp. 2295–2308, Nov. 2000.

[27] Y. Xuan and B. Coifman, "Lane change maneuver detection from probe vehicle GPS data," in Proceedings of the International Conference on Intelligent Transportation System, Sep. 2006, pp. 624–629.

[28] P. Angkititrakul, R. Terashima, and T. Wakita, "On the use of stochastic driver behavior model in lane departure warning," *IEEE Transactions on Intelligent Transportation Systems*, vol. 12, no. 1, pp. 174–183, Mar. 2011.

[29] S. Ammoun, F. Nashashibi, and C. Laurgeau, "An analysis of the lane changing maneuver on roads: the contribution of inter-vehicle cooperation via communication," in Proceedings of the Intelligent Vehicles Symposium, Jun. 2007, pp. 1095–1100.

[30] L. Li, F. Y. Wang, and Y. Zhang, "Cooperative driving at lane closures," in Proceedings of the Intelligent Vehicles Symposium, Jun. 2007, pp. 1156–1161.

[31] H. Park and B. Smith, "Investigating benefits of intellidrive in freeway operations: Lane changing advisory case study," *Journal of Transportation Engineering*, vol. 138, no. 9, pp. 1113–1122, Sep. 2012.

[32] A. Bohm, M. Jonsson, and E. Uhlemann, "Adaptive cooperative awareness messaging for enhanced overtaking assistance on rural roads," in Proceedings of the Vehicular Technology Conference, Sep. 2011, pp. 1–5.

[33] P. Lytrivis, G. Thomaidis, M. Tsogas, and A. Amditis, "An advanced cooperative path prediction algorithm for safety applications in vehicular networks*," IEEE Transactions on Intelligent Transportation Systems*, vol. 12, no. 3, pp. 669–679, Sep. 2011.

[34] M. Ardelt, C. Coester, and N. Kaempchen, "Highly automated driving on freeways in real traffic using a probabilistic framework," *IEEE Transactions on Intelligent Transportation Systems*, vol. 13, no. 4, pp. 1576–1585, Dec. 2012.

[35] R. Horowitz, X. Sun, and C.-W. Tan, "An efficient lane change maneuver for platoons of vehicles in an automated highway system," California PATH Research Report, Tech. Rep., 2004.

[36] H.C.-H. Hsu and A. Liu, "Platoon lane change maneuvers for automated highway systems," in Proceedings of the Conference on Robotics, Automation and Mechatronics, Dec. 2004, pp. 780–785.

[37] ——, "Kinematic design for platoon-lane-change maneuvers," *IEEE Transactions on Intelligent Transportation Systems*, vol. 9, no. 1, pp. 185–190, Jan. 2008.

[38] M. Hilscher, S. Linker, E.-R. Olderog, and A. Ravn, "An abstract model for proving safety of multi-lane traffic manoeuvres," in Proceedings of the International Conference on Formal Methods and Software Engineering, Oct. 2011, pp. 404–419.

[39] R. Hall and C. Caliskan, "Design and evaluation of an automated highway system with optimized lane assignment," *Transportation Research Part C: Emerging Technologies*, vol. 7, no. 1, pp. 1–15, Feb. 1997.

[40] R. Hall and C. Chinan, "Vehicle sorting for platoon formation: Impacts on highway entry and throughput," *Transportation Research Part C: Emerging Technologies*, vol. 13, no. 5, pp. 405–420, Oct. 2005.

[41] D. Ramaswamy, J. Medanic, W. Perkins, and R. Benekohal, "Lane assignment on automated highway systems," *IEEE Transactions on Vehicular Technology*, vol. 46, no. 3, pp. 755–769, Aug. 1997.

[42] K. Kim, D.I. Cho, and J. Medanic, "Lane assignment using a genetic algorithm in the automated highway systems," in Proceedings of the International Conference on Intelligent Transportation System, Sep. 2005, pp. 540–545.

[43] P. Gipps, "A behavioural car-following model for computer simulation," *Transportation Research Part B: Methodological*, vol. 15, no. 2, pp. 105–111, Apr. 1981.

[44] A. Bohm and M. Jonsson, "Supporting real-time data traffic in safety-critical vehicle-to-infrastructure communication," in Proceedings of the Conference Local Computer Networks, Oct. 2008, pp. 614–621.

[45] J. Santa, A. Gómez-Skarmeta, and M. Sánchez-Artigas, "Architecture and evaluation of a unified V2V and V2I communication system based on cellular networks," *Computer Communications*, vol. 31, no. 12, pp. 2850–2861, Jul. 2008.

[46] J. Neades and R. Ward, "Swerves and lane changes," Professional Development in Road Accident Investigation Course. [Online]. Available:
http://www.mathtech.co.uk/Downloads/Swerves\%20and\%20Lane\%20Changes.pdf