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PHYSICS-INFORMED
MACHINE LEARNING
FOR SEISMIC SITE
CHARACTERIZATION FROM
LIMITED RECORDINGS



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16. Abstract Reliable knowledge of subsurface soil properties is needed to predict earthquake motions and assess seismic hazards. However, these properties are often uncertain because site investigations provide only limited measurements. Conventional seismic inversion methods can estimate soil properties from recorded ground motions, but they are computationally demanding and may depend strongly on the assumed starting model. This project developed a physics-informed neural network framework that combines seismic observations with the physical equations governing wave propagation. The framework was designed to recover unknown input motions, estimate shear-wave velocity, and account for energy loss through material damping. It was evaluated using four- and six-layer soil profiles derived from published site data, followed by an application using field recordings from the Treasure Island Downhole Array. The framework recovered base motions from single surface recordings with errors below 6% and reproduced the main features of the reference shear-wave velocity profiles using only two recordings. The results also demonstrated the feasibility of estimating velocity and damping simultaneously, although damping and thin soil layers were more difficult to resolve. The field application captured the overall variation of shear-wave velocity with depth and the principal material contrasts. For transportation infrastructure, poorly characterized low-stiffness zones and other subsurface anomalies can contribute to roadway deformation and subsidence, making improved subsurface characterization important for proactive asset management. These findings show that physics-informed machine learning can support seismic deconvolution and subsurface characterization when measurement data are limited. Further development could provide engineers with an additional tool for site-response analysis, earthquake-hazard assessment, and geotechnical design.			
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ABSTRACT

Reliable knowledge of subsurface soil properties is needed to predict earthquake motions and assess seismic hazards. However, these properties are often uncertain because site investigations provide only limited measurements. Conventional seismic inversion methods can estimate soil properties from recorded ground motions, but they are computationally demanding and may depend strongly on the assumed starting model. This project developed a physics-informed neural network framework that combines seismic observations with the physical equations governing wave propagation. The framework was designed to recover unknown input motions, estimate shear-wave velocity, and account for energy loss through material damping. It was evaluated using four- and six-layer soil profiles derived from published site data, followed by an application using field recordings from the Treasure Island Downhole Array. The framework recovered base motions from single surface recordings with errors below 6% and reproduced the main features of the reference shear-wave velocity profiles using only two recordings. The results also demonstrated the feasibility of estimating velocity and damping simultaneously, although damping and thin soil layers were more difficult to resolve. The field application captured the overall variation of shear-wave velocity with depth and the principal material contrasts. For transportation infrastructure, poorly characterized low-stiffness zones and other subsurface anomalies can contribute to roadway deformation and subsidence, making improved subsurface characterization important for proactive asset management. These findings show that physics-informed machine learning can support seismic deconvolution and subsurface characterization when measurement data are limited. Further development could provide engineers with an additional tool for site-response analysis, earthquake-hazard assessment, and geotechnical design.

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LIST OF KEY TERMS

Term	Definition
1D	One-dimensional
4L	Four-layer soil profile
6L	Six-layer soil profile
ABC	Absorbing boundary condition
BC	Boundary condition
CNN	Convolutional neural network
DL	Deep learning
FDM	Finite-difference method
FFNN	Fully connected feed-forward neural network
FWI	Full-waveform inversion
IC	Initial condition
L_2 norm	Euclidean norm used to quantify waveform error
MAPE	Mean absolute percentage error
MASW	Multichannel analysis of surface waves
PDE	Partial differential equation
PIML	Physics-informed machine learning
PINNs	Physics-informed neural networks
PINNs-FWI	Physics-informed neural networks–based full-waveform inversion
SASW	Spectral analysis of surface waves
SH wave	Horizontally polarized shear wave
TIDA	Treasure Island downhole array
C	Constitutive or material-rigidity operator
f	Body force per unit mass (m/s^2)
G	Shear modulus (Pa)
H	Total depth of the soil column (m)
t	Time (s)
u	Horizontal displacement (m)
V_p	Compressional-wave velocity (m/s)
V_s	Shear-wave velocity (m/s)
z	Depth coordinate (m)
γ	Shear strain (dimensionless)
η	Kelvin–Voigt viscosity coefficient ($\text{Pa} \cdot \text{s}$)
ρ	Mass density (kg/m^3)
σ	Cauchy stress tensor (Pa)
τ	Shear stress (Pa)

Term	Definition
ψ	Shear-wave potential
ζ	Damping ratio (dimensionless; reported as a percentage)
ω_0	Reference circular frequency (rad/s)
Θ	Trainable neural-network parameters
$\mathcal{L}$	Total PINN loss function

EXECUTIVE SUMMARY

Uncertainty in subsurface material properties presents a major challenge for seismic-response prediction and geotechnical site characterization. Conventional full-waveform inversion (FWI) uses complete seismic records to estimate these properties but can be computationally demanding and sensitive to the initial model, measurement quality, and coupling between parameters. This study develops a physics-informed neural network-based full-waveform inversion (PINNs-FWI) framework for one-dimensional elastic and viscoelastic layered soil deposits.

The framework incorporates seismic observations, governing wave equations, and initial and boundary conditions into neural-network training. The elastic formulation estimates shear-wave velocity, while the viscoelastic formulation uses a Kelvin-Voigt model to estimate shear-wave velocity and damping ratio simultaneously. Representative four- and six-layer models were developed from subsurface velocity profiles reported for sites across the United States. These models were used to evaluate base-motion deconvolution and material-property inversion under different levels of subsurface complexity.

The framework recovered the base motions for the elastic and viscoelastic cases with normalized errors below 6%. It also reproduced the principal features of the reference velocity profiles, although thin layers and sharp interfaces were more difficult to resolve. The viscoelastic results showed that damping-ratio estimation was less accurate than velocity estimation because the recorded wavefield is less sensitive to damping and because velocity and damping can trade off during inversion.

The framework was further evaluated using field recordings from the Treasure Island Downhole Array. Using the measured surface recording together with a model-derived response at a depth of 80 m, the inversion captured the general increase in shear-wave velocity with depth and the principal velocity contrasts. For transportation applications, these capabilities provide a foundation for identifying low-stiffness zones associated with soft soils, loose deposits, voids, and other subsurface conditions that may contribute to roadway deformation or subsidence. Overall, the results demonstrate the potential of PINNs-FWI for seismic deconvolution and subsurface characterization using limited measurements, while highlighting the need for further improvements in damping estimation, thin-layer resolution, and applications involving more complex site conditions.

1. INTRODUCTION AND BACKGROUND

1.1 Project Motivation

Large uncertainties in natural geomaterials constrain the practical application of theoretical predictive models to real-world problems in geoscience and geoengineering. Part of this uncertainty is intrinsic to natural geological processes, including variations in deposition, progressive weathering, and changes in material behavior caused by environmental conditions. Epistemic uncertainty also results from limited site investigations, imperfect measurements and interpretations, and insufficient information about subsurface stratigraphy and boundary conditions (Phoon and Kulhawy 1999; Schweiger and Peschl 2005; Teague and Cox 2016). To address the epistemic challenge, geophysical techniques based on seismic wave propagation have increasingly become important tools for improving subsurface characterization. By analyzing how seismic waves respond to changes in material properties and geological layering, these techniques can provide information that reduces uncertainty associated with incomplete knowledge of subsurface conditions.

Non-invasive methods such as spectral analysis of surface waves (SASW) (Stokoe et al. 1994) and multichannel analysis of surface waves (MASW) (Park et al. 1999) are widely favored for their cost-effectiveness and straightforward field implementation for near-surface characterization (Socco et al. 2010; Kallivokas et al. 2013). However, these methods commonly assume horizontally layered media and use only selected features of the measured wavefield, such as dispersion curves. These simplifications can limit their ability to characterize irregular, laterally variable, or strongly heterogeneous subsurface conditions (Zhang et al. 2024). FWI (Tarantola 1984; Virieux et al. 2014) offers a more comprehensive approach by utilizing the complete recorded seismic wavefield and can provide higher-resolution estimates of subsurface properties. This method iteratively updates subsurface material models by minimizing the discrepancy between observed data and numerically simulated responses. Nevertheless, FWI is highly sensitive to the initial model and can be affected by cycle skipping, local minima, sparse observations, measurement noise, and coupling between different material parameters (Bunks et al. 1995; Virieux and Operto 2009; Qu et al. 2019; Gao et al. 2021). Furthermore, multiparameter FWI remains challenging when the target parameters have strongly different sensitivities and various orders of magnitude, which can produce severe parameter crosstalk, trade-offs, and unstable updates, particularly for near-surface site characterization, such as jointly estimating velocity and damping ratio, which often differ substantially in magnitude (Operto et al. 2013; Prieux et al. 2013).

In response to the challenges of traditional FWI, recent years have seen growing interest in deep learning (DL) approaches, which offer computationally efficient alternatives for seismic inversion and wavefield prediction (Adler et al. 2021). These methods use seismic observations to infer subsurface structures and generate robust starting models that are less susceptible to local minima during optimization. For example, Vantassel et al. (2022) employed convolutional neural networks (CNNs) with time–distance-domain data, whereas Abbas et al. (2023) used frequency–velocity-domain inputs to construct two-dimensional near-surface velocity models. Once trained over specified parameter ranges, these networks can rapidly estimate starting velocity models from seismic observations, potentially reducing the computational burden that has historically limited the application of FWI in geotechnical engineering. In addition, DL frameworks have

been developed to simultaneously predict multiple subsurface parameters (e.g., V_p , V_s , and density, or other coupled properties) from seismic data, providing a practical route for multi-parameter inversion beyond single-parameter velocity imaging (Kazei et al. 2021; Wu et al. 2025). However, purely data-driven models require extensive training datasets and may generalize poorly to subsurface conditions that are not represented in the training datasets (Moseley et al. 2020b; Cao et al. 2022; Li et al. 2023). Because conventional supervised-learning methods do not explicitly enforce the governing physical laws, their predictions may also be physically inconsistent (Karniadakis et al. 2021; Wu et al. 2023; Chai et al. 2023).

Physics-informed neural networks (PINNs) offer a promising alternative by incorporating the governing differential equations, initial conditions, and boundary conditions directly into neural-network training (Raissi et al. 2019; Karniadakis et al. 2021; Cuomo et al. 2022). PINNs can address forward and inverse problems within a common computational framework and can treat unknown material properties, external excitations, and boundary conditions as trainable variables (Rasht-Behesht et al. 2022; Ren et al. 2023; Kapoor et al. 2024). However, their application to high-resolution geotechnical site characterization remains challenging because standard neural networks exhibit spectral bias, preferentially learning smooth, low-frequency features and potentially failing to resolve oscillatory wavefields and sharp impedance contrasts. These difficulties become more pronounced in heterogeneous media and multiparameter inversion, where properties such as wave velocity and damping may have different sensitivities, numerical scales, and effects on the wavefield.

A unified physics-informed inversion framework is therefore needed to model seismic-wave propagation while identifying subsurface interfaces and estimating multiple material properties from limited observations. Such a framework must capture sharp material discontinuities and low-amplitude, oscillatory wave components while accommodating differences in the sensitivities and numerical scales of parameters such as shear-wave velocity and damping ratio. Addressing these challenges through a PINNs-FWI framework could improve the accuracy and stability of near-surface characterization while reducing dependence on extensive labeled datasets and highly accurate initial models. For transportation infrastructure, improved subsurface characterization can support the identification of low-stiffness zones associated with loose deposits, soft soils, voids, and other conditions that may contribute to roadway deformation or subsidence. Accordingly, this study develops and evaluates the underlying inversion methodology for estimating shear-wave velocity and damping properties from limited seismic observations. Extending the framework to detect localized two- and three-dimensional roadway anomalies is reserved for future research.

1.2 Research, Objectives, and Tasks

To address the limitations of conventional full-waveform inversion and purely data-driven methods, this research aims to develop a unified PINNs-FWI framework for seismic-wave prediction and subsurface characterization in one-dimensional layered geomaterials. The framework addresses forward and inverse problems using limited observations while enforcing the governing equations, constitutive relationships, and relevant initial and boundary conditions. Particular emphasis is placed on representing sharp subsurface interfaces, resolving reflected and transmitted wave components, and estimating material properties with different sensitivities and numerical scales, including shear-wave velocity and damping ratio.

The original project comprised three objectives: (1) generating seismic wavefields for representative shallow subsurface conditions, (2) developing a PINNs-FWI framework for material-property inversion, and (3) evaluating the framework using field seismic recordings. As the research progressed, the second objective was refined and divided into separate elastic and viscoelastic analyses. This refinement distinguishes single-parameter shear-wave velocity inversion from multiparameter inversion of shear-wave velocity and damping ratio, both of which are relevant to geotechnical site characterization and dynamic soil response. Accordingly, the three original objectives are presented through four technical tasks: Objective 1 corresponds to Task 1, Objective 2 is addressed by Tasks 2 and 3, and Objective 3 corresponds to Task 4. These tasks are described below.

Task 1: Development of a Representative Numerical Simulation Dataset

Thirty published subsurface shear-wave velocity profiles from sites across the United States were reviewed to establish representative ranges of layer thickness and shear-wave velocity. From these data, simplified four- and six-layer profiles were developed to represent different levels of subsurface complexity. Validated numerical solvers were then used to generate reference elastic and viscoelastic wavefields for developing and evaluating the proposed PINNs-FWI framework under controlled conditions.

Task 2: One-Dimensional Elastic Wave Analysis

Develop a PINNs-FWI framework for seismic-wave propagation in one-dimensional elastic layered media. Forward analyses are conducted to predict displacement, velocity, and stress responses under prescribed material properties, input motions, and boundary conditions. The framework is then applied to inverse analysis to estimate unknown elastic properties from limited seismic observations.

Task 3: One-Dimensional Viscoelastic Wave Analysis and Multiparameter Inversion

Extend the elastic formulation to one-dimensional viscoelastic layered media to account for material damping and seismic-wave attenuation. The framework is used to predict attenuated wavefields and simultaneously estimate multiple unknown material properties, particularly wave velocity and damping ratio, from limited seismic observations. This multiparameter inversion addresses challenges associated with differences in parameter sensitivity and numerical scale, as well as potential trade-offs between stiffness- and damping-related effects on the observed wavefield. The task evaluates the ability of the PINNs-FWI framework to distinguish these effects and reliably characterize the viscoelastic behavior of layered geomaterials.

Task 4: Model Verification and Performance Evaluation

Verify the PINNs-FWI framework by comparing its forward predictions and inversion results with reference numerical solutions and evaluating the field-informed TIDA application. Performance is evaluated in terms of wavefield reconstruction accuracy, material-property estimation, and the ability to recover key subsurface features under different levels of model complexity.

1.3 Report Overview

The report is organized around two main technical components: one-dimensional elastic analysis and one-dimensional viscoelastic analysis. Section 1 introduces the motivation for physics-informed seismic-wave modeling and defines the research objectives and tasks. Section 2 reviews applications of physics-informed machine learning to forward wave modeling and seismic inversion. Section 3 presents the governing elastic and viscoelastic wave equations, initial and boundary conditions, and the architecture and training formulation of the proposed PINNs-FWI framework.

The first technical component addresses one-dimensional elastic-wave problems. It includes base-motion deconvolution for known layered profiles, shear-wave velocity inversion using limited recordings, and application to field measurements from the Treasure Island Downhole Array (TIDA). The second component extends the framework to one-dimensional viscoelastic media using a Kelvin-Voigt constitutive model. It evaluates viscoelastic deconvolution and multiparameter inversion for shear-wave velocity and damping ratio. Sections 4 and 5 describe the numerical and field investigations and discuss the corresponding results. Section 6 summarizes the study's findings, limitations, and implications for geotechnical site characterization.

2. LITERATURE REVIEW

2.1 Physics-informed Machine Learning in Wave Modeling

Table 1. Review of physics-informed machine learning studies for seismic-wave modeling.

Reference	Objectives
Karimpouli and Tahmasebi (2020)	Evaluate the Gaussian-process and PINNs methods for solving the 1D seismic-wave equation with limited data.
Moseley et al. (2020a)	Develop a PINNs for modeling 2D acoustic-wave propagation in spatially varying velocity models.
Song and Wang (2023)	Develop a Fourier-feature PINNs for predicting wavefields at multiple frequencies and source locations.
Ren et al. (2024)	Model seismic-wave propagation in semi-infinite domains under different source and material conditions.
Ding et al. (2025)	Improve time-domain PINNs simulation of high-frequency wave propagation in non-smooth velocity models.
Zou et al. (2025)	Accelerate PINNs training by combining finite-difference and automatic differentiation derivatives.
Zhu et al. (2025)	Improve 2D elastic-wave simulation using adaptive loss weighting and transfer learning.

Physics-informed machine learning (PIML) provides an alternative approach to seismic-wave modeling by incorporating governing equations and initial and boundary conditions into neural-network training. Karimpouli and Tahmasebi (2020) showed that Gaussian processes and PINNs could solve the one-dimensional seismic-wave equation using limited observations. Moseley et al. (2020a) extended PINNs to the two-dimensional acoustic-wave equation for homogeneous, layered, and spatially variable velocity models. By including source location as a network input, their model predicted wavefields for different source positions without retraining for each simulation.

Later studies addressed boundary reflections, long-duration simulations, and spectral bias. Ren et al. (2024) developed SeismicNet by incorporating absorbing boundary conditions, temporal domain decomposition, and variable source locations into a forward-modeling framework. Song and Wang (2023) introduced Fourier-feature mappings to improve the representation of multifrequency wavefields and allow simulations at different source locations. Ding et al. (2025) extended Fourier-feature PINNs to time-domain wave propagation in non-smooth velocity models and incorporated paraxial absorbing boundary conditions to reduce artificial reflections.

Other studies focused on training convergence and model reuse. Zhu et al. (2025) combined adaptive loss weighting with transfer learning for two-dimensional elastic-wave simulations involving changes in source location and material properties. Zou et al. (2025) used finite-difference derivatives during the initial training stage and automatic differentiation during later refinement, reducing the number of iterations required for convergence in two- and three-dimensional examples. These studies indicate that PIML can represent seismic-wave propagation

under a range of source and material conditions. However, most existing studies use synthetic, acoustic, or elastic models. Applications involving viscoelastic attenuation, layered soil profiles, weak reflected waves, and field-derived velocity structures remain limited.

2.2 Physics-informed Machine Learning in Seismic Inversion

Table 2. Review of physics-informed machine learning studies for seismic inversion.

Reference	Objectives
Rasht-Behesht et al. (2022)	Apply PINNs to 2D acoustic FWI using limited seismic observations and physical constraints.
Yang and Ma (2023)	Develop an unsupervised adversarial FWI method with reduced sensitivity to noise and initial models.
Lu et al. (2024)	Perform multiparameter elastic FWI by incorporating geological and borehole prior information.
Zhang et al. (2023)	Simultaneously estimate velocity and density using surface seismograms and well-log observations.
Mohammadi et al. (2024)	Evaluate PINNs-FWI for elastic deconvolution and geotechnical subsurface characterization.
Ren et al. (2023)	Estimate the material distribution of a 1D layered elastic medium from sparse surface measurements.

PIML has also been applied to seismic inversions by treating unknown subsurface properties as trainable variables together with the neural-network parameters. Rasht-Behesht et al. (2022) applied PINNs to two-dimensional acoustic full-waveform inversion using limited wavefield observations, absorbing boundary conditions, and prior information about the subsurface. The method recovered velocity anomalies in the numerical cases considered, although the reconstruction of sharp interfaces and complex velocity structures remained difficult.

Subsequent studies extended PIML inversion to multiple material parameters and different objective functions. Zhang et al. (2023) embedded first-order acoustic-wave equations into a PINNs framework and combined surface seismograms with well-log data to estimate velocity and density simultaneously. Their method did not require an accurate initial material model and was tested with both noise-free and noisy data. Yang and Ma (2023) proposed FWIGAN, an unsupervised framework that uses a Wasserstein-based measure to compare simulated and observed seismograms. The method reduced sensitivity to the initial model and data noise in the tested examples but required more memory and computation than conventional FWI.

For layered and geotechnical applications, Ren et al. (2023) used two networks to reconstruct the wavefield and estimate the material distribution in a one-dimensional semi-infinite elastic medium. Mohammadi et al. (2024) applied PINNs-FWI to deconvolution and site characterization using synthetic data and recordings from the TIDA. The method recovered the general shear-wave velocity structure but smoothed some layer boundaries. Lu et al. (2024) incorporated geological information, borehole velocity profiles, and empirical relationships between P- and S-wave velocities into a physics-informed recurrent neural network for multiparameter elastic FWI.

These studies demonstrate that PIML can combine seismic measurements, governing equations, and prior subsurface information within a single inversion formulation. Remaining challenges include recovering discontinuous material profiles, processing sparse and noisy field measurements, and estimating parameters with different sensitivities and numerical scales. In particular, the simultaneous inversion of shear-wave velocity and damping ratio in layered viscoelastic media has received limited attention.

Mohammadi et al. (2024) presented the initial elastic PINNs-FWI formulation for seismic deconvolution and site characterization, including an application to the TIDA. The present project builds on that work by expanding the evaluation to four- and six-layer profiles with different levels of subsurface complexity and extending the framework to viscoelastic deconvolution and joint inversion of shear-wave velocity and damping ratio. The TIDA analysis is retained as a field-based evaluation of the elastic formulation, whereas the viscoelastic and multiparameter investigations represent the principal extensions developed in this study.

3. METHODOLOGY

3.1 Wave Equations

The propagation of stress waves in a solid medium is governed by the balance of linear momentum, expressed in terms of the displacement vector $\mathbf{u}$ and the Cauchy stress tensor $\boldsymbol{\sigma}$:

$$\nabla \cdot \boldsymbol{\sigma} + \mathbf{f} = \rho \mathbf{u}_{tt}, \quad (1)$$

where ρ is the mass density, $\mathbf{f}$ is the body-force vector per unit mass, and the subscript tt denotes the second derivative with respect to time. Constitutive equations can be used to express the stress tensor in terms of the displacement vector:

$$\boldsymbol{\sigma} = \mathbf{C}(\nabla \mathbf{u} + \mathbf{u} \nabla)/2 \quad (2)$$

For a one-dimensional soil column subjected to vertically propagating horizontally polarized shear (SH) waves, the deformation is represented by a single horizontal displacement component $u(z, t)$. Equation (1), therefore, reduces to:

$$\frac{\partial \tau}{\partial z} + f = \rho u_{tt}, \quad (3)$$

where $\tau(z, t)$ is the shear stress. For a linear elastic medium, the constitutive relationship is

$$\tau = G\gamma, \quad \gamma = u_z \quad (4)$$

where γ is the shear strain. In the absence of body forces, substituting Equation (4) into Equation (3) gives the one-dimensional elastic shear-wave equation:

$$u_{tt} = V_s^2 u_{zz}, \quad V_s = \sqrt{\frac{G}{\rho}} \quad (5)$$

The wavefield may alternatively be represented using the shear-wave potential $\psi(z, t)$, with the physical displacement defined as

$$u(z, t) = -\psi_z(z, t) \quad (6)$$

Under this representation, the elastic wave equation becomes

$$\psi_{tt} = V_s^2 \psi_{zz}. \quad (7)$$

To account for material damping and wave attenuation, the elastic formulation is extended using a Kelvin–Voigt-type viscoelastic constitutive model (Assimaki et al. 2012; Poursartip et al. 2020):

$$\tau = G\gamma + \eta\gamma_t, \quad \gamma = u_z, \quad (8)$$

where η is the viscosity coefficient. Consequently, the shear stress can be written as

$$\tau = Gu_z + \eta u_{zt}. \quad (9)$$

Substituting Equations (9) and (6) into Equation (3) yields the one-dimensional viscoelastic shear-wave equation. Furthermore, the viscosity coefficient is parameterized in terms of an effective damping ratio ζ at a reference circular frequency ω_0 :

$$\eta = \frac{2G\zeta}{\omega_0}. \quad (10)$$

The reference frequency ω_0 is selected to be consistent with the dominant frequency content of the input motion, such as the central frequency of a Ricker wavelet. Accordingly, ζ should be interpreted as the effective damping ratio represented by the Kelvin–Voigt model at the selected reference frequency ω_0 , rather than as a frequency-independent material damping ratio. Using Equation (10) in the viscoelastic wave equation derived from Equations (3), (6), and (9) gives

$$\psi_{tt} = V_s^2 \psi_{zz} + \frac{2\zeta V_s^2}{\omega_0} \psi_{zzt}. \quad (11)$$

Equation (7) is recovered from Equation (11) when $\zeta = 0$, allowing the elastic and viscoelastic problems to be addressed within a consistent mathematical framework.

Appropriate initial and boundary conditions are required to obtain a unique solution. Unless an initial disturbance is explicitly prescribed, the soil column is initially at rest:

$$u(z, 0) = 0, \quad u_t(z, 0) = 0. \quad (12)$$

At the ground surface, $z = 0$, a traction-free boundary condition is imposed:

$$\tau(0, t) = 0. \quad (13)$$

At the base of the soil column, $z = H$, either a prescribed motion or a compliant absorbing boundary is applied, depending on the analysis. For an absorbing base, the first-order Lysmer–Kuhlemeyer condition is expressed as Lysmer and Kuhlemeyer (1969):

$$\tau(H, t) + \rho V_s u_t(H, t) = 2\rho V_s S(t). \quad (14)$$

Here, $S(t)$ denotes the incident shear-wave particle-velocity history prescribed at the compliant base. Setting $S(t) = 0$ gives the unforced absorbing boundary condition. Subject to these initial and boundary conditions, the proposed framework considers forward, deconvolution, and inverse-medium problems. In the forward problem, the seismic response is predicted from known input motion and material profiles. In the deconvolution problem, the base motion is recovered using a surface recording and a known subsurface profile. In the inverse-medium problem, surface and borehole recordings are used to estimate the unknown material properties. The elastic formulation focuses on estimating the shear-wave velocity profile $V_s(z)$, whereas the viscoelastic formulation performs multiparameter inversion for both $V_s(z)$ and $\zeta(z)$.

3.2 Physics-Informed Neural Network Framework

This section presents the architecture of the proposed PINNs framework, which serves to predict wavefields in the forward simulation and to extract seismic wave velocities in the inverse part.

The mathematical formulation of the fully connected feed-forward neural network (FFNN) is first summarized.

3.2.1 Neural Network – Mathematical Setup

Following the original work of Raissi et al. (2019) and its extension to the wave propagation problem (Rasht-Behesht et al. 2022), we consider a fully connected FFNN with an input layer comprising the spatiotemporal coordinates z and t , L hidden layers, and an output layer representing shear wave potential (ψ) in forward modeling. The inverse network maps depth to shear-wave velocity for elastic cases, and to shear-wave velocity (V_s) and damping ratio (ζ) for viscoelastic cases. In this FFNN, the l^{th} hidden layer of width n^l (number of neurons) takes the output of the previous layer $\mathbf{x}^{l-1}$ and generates input for the next layer $\mathbf{x}^l$ through the following mixed transformation:

$$\mathbf{x}^l = H(\mathbf{x}^{l-1}) = \alpha(\mathbf{w}^l \mathbf{x}^{l-1} + \mathbf{b}^l) \quad \text{for } 0 < l < L + 1 \quad (15)$$

where $\mathbf{w}^l$ and $\mathbf{b}^l$ denote the trainable matrix of network weights and vector of bias terms for the l^{th} layer that forms the linear transformation inside parentheses on the right-hand side of Equation (15), and α is a neuron-wise nonlinear activation function that updates the output of the linear transformation. In this study, we use hyperbolic tangent ($\tanh$) as the activation function for all neural networks.

3.3 PINNs Formulation

The neural networks described in the previous section can approximate unknown variables (wavefields or material properties) when trained with adequate labeled data. To reduce this data dependency, PINNs incorporate physical laws—governing equations and boundary conditions—directly into the solution process. The components of these differential equations, including spatial and temporal derivatives, are integrated into neural networks through automatic differentiation using the chain rule (Baydin et al. 2018). Unknown (trainable) parameters of the problem consist of all the weights and biases of the neural networks $\Theta = (\mathbf{w}, \mathbf{b})$. PINNs can learn the optimal values of weights and biases ($\mathbf{w}^*, \mathbf{b}^*$) by minimizing the loss function:

$$\Theta^* = \arg \min \{ \mathcal{L}(\Theta) \} \quad (16)$$

where the loss function, $\mathcal{L}(\Theta)$, contains several residual terms corresponding to the governing equation (R_{PDE}), initial conditions (ICs) (R_{IC}), boundary conditions (BCs) (R_{BC}), and observation data (R_{OBS}):

$$\mathcal{L}(\Theta) = \beta_{\text{OBS}} R_{\text{OBS}} + \beta_{\text{PDE}} R_{\text{PDE}} + \beta_{\text{BC}} R_{\text{BC}} + \beta_{\text{IC}} R_{\text{IC}} \quad (17)$$

Here, the β_i terms are the loss weights, used to balance the contribution of each component to the total loss. The first term in Equation (17) represents the standard residual used in data-driven methods, expressed as the difference between observed and computed data measured by a specific norm. Using the mean squared error and considering N_{OBS} spatiotemporal collocation points (z_i, t_i) to enforce data fidelity, the observation residual term is defined as follows:

$$R_{\text{OBS}} = \frac{1}{N_{\text{OBS}}} \sum_{i=1}^{N_{\text{OBS}}} |\mathbf{u}_{\text{OBS}}(z_i, t_i) - \tilde{\mathbf{u}}(z_i, t_i)|^2 \quad (18)$$

In our problem, the observed data refer to the actual/synthetic wavefields (seismograms) recorded on the ground surface or within boreholes. The final three residual terms in Equation (17) represent the physical constraints in PINNs, derived from the governing equations and ICs/BCs. For instance, the PDE residual is calculated by enforcing the Navier-Cauchy wave equation at a finite set of N_{PDE} collocation points distributed throughout the space-time domain, expressed as:

$$R_{\text{PDE}} = \frac{1}{N_{\text{PDE}}} \sum_{i=1}^{N_{\text{PDE}}} |\nabla \tilde{\sigma} + \mathbf{f} - \rho \tilde{\mathbf{u}}_{tt}|^2 \quad (19)$$

Similar expressions can be formulated to calculate the residual terms for the equations defining the initial and boundary conditions, as defined in Section 3.1, at their respective spatiotemporal collocation points. We randomly sample training points across the appropriate space-time domains to evaluate these various residual terms.

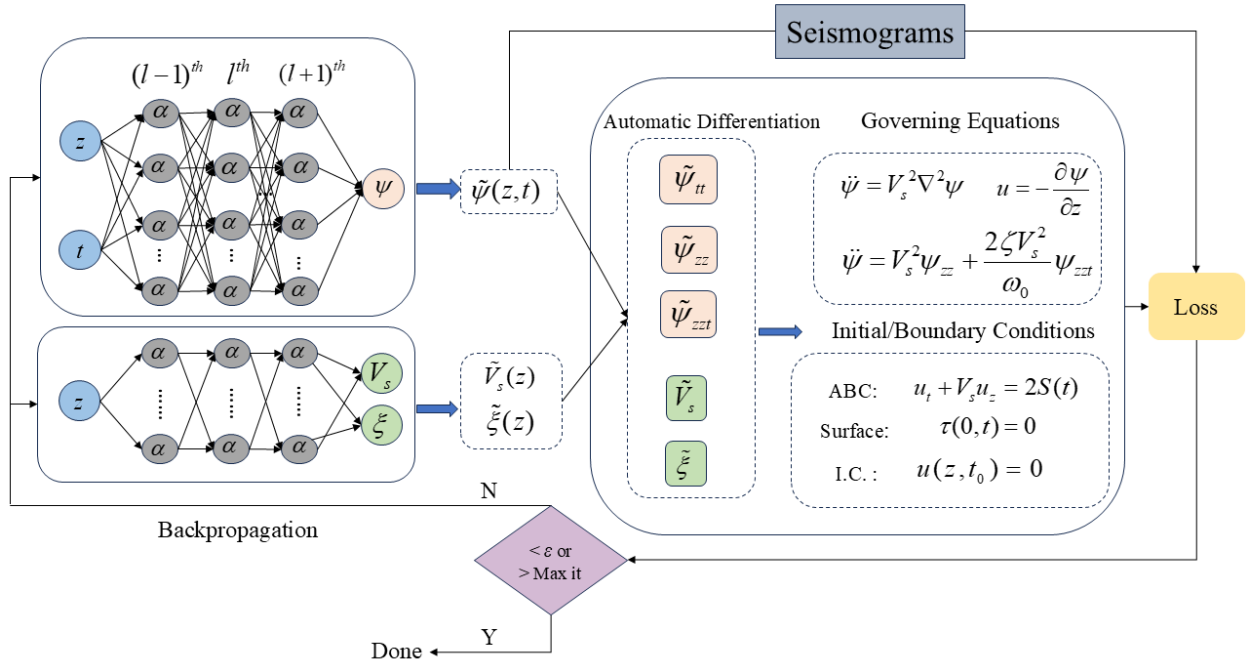


Figure 1. Schematic of the proposed PINNs-FWI framework for forward and inverse analysis

Figure 1 illustrates the schematic diagram of our proposed PINNs framework for solving forward and inverse elastic wave equations. Following established approaches (Rasht-Behesht et al. 2022; Shukla et al. 2022), we implement two separate neural networks: one for the forward problem (predicting wavefields) and another for the inverse problem (estimating wave velocities). This separation enhances overall performance and interpretability while providing flexibility to address problems either jointly or individually. The forward network employs a larger architecture to effectively capture the complex spatiotemporal variations in wavefields. In contrast, the inverse network utilizes a more compact structure since velocity profiles vary only

spatially with fewer fluctuations. The inverse network maps the spatial coordinates to the corresponding subsurface material properties. The number of output variables depends on the material model considered. For an elastic problem, the network has a single output representing the shear-wave velocity. For a viscoelastic problem, the network has two outputs representing the shear-wave velocity and damping ratio. We then employ automatic differentiation to incorporate the wave equation into the loss function, which is optimized using the Adam stochastic gradient descent algorithm (Kingma and Ba 2014). Training terminates upon reaching either the specified error threshold (ε) or the maximum iteration limit.

3.4 Evaluation Metrics

Two error metrics are used to evaluate the predicted waveforms and inverted material profiles. Waveform accuracy is quantified using the normalized L_2 error:

$$e_{L_2} = \frac{\|\hat{\mathbf{y}} - \mathbf{y}\|_2}{\|\mathbf{y}\|_2} \times 100\%,$$

where $\mathbf{y}$ and $\hat{\mathbf{y}}$ denote the reference and predicted waveforms, respectively. The error is calculated using the original, unscaled signals; waveform normalization is applied only for visualization.

Material-profile accuracy is evaluated using the mean absolute percentage error (MAPE):

$$\text{MAPE} = \frac{100\%}{N_z} \sum_{i=1}^{N_z} \left| \frac{\hat{m}_i - m_i}{m_i} \right|,$$

where m_i and $\hat{m}_i$ are the reference and estimated material properties at the i th depth point, respectively, and N_z is the number of depth points used in the evaluation. MAPE is calculated separately for the shear-wave velocity and damping-ratio profiles.

4. NUMERICAL AND FIELD INVESTIGATIONS

To evaluate the performance of the proposed PINNs-FWI framework for elastic and viscoelastic wave deconvolution and material-property inversion, several one-dimensional computational experiments are conducted, as described in this section.

One-dimensional viscoelastic multiparameter inversion presents two principal challenges related to parameter identifiability and subsurface resolution. First, the simultaneous estimation of shear-wave velocity, V_s , and damping ratio, ζ , is often ill-conditioned. The target parameters differ substantially in numerical scale: V_s commonly ranges from $O(10^2)$ to $O(10^3)$ m/s, whereas ζ is dimensionless and typically of order $O(10^{-2})$ (Tables 3 and 4). More importantly, the recorded wavefield has different sensitivities to velocity and damping. Velocity primarily controls wave travel time and phase, whereas damping influences amplitude decay and frequency content. These differences can produce parameter trade-offs, increase non-uniqueness, and reduce inversion accuracy relative to single-parameter formulations (Operto et al. 2013; Prioux et al. 2013; Gao et al. 2021; Fathi et al. 2015). Second, resolving layered subsurface structures remains difficult. Standard regularization methods often impose smoothness constraints that blur lithological boundaries and suppress the high-wave number components required to represent abrupt, step-like material contrasts. Consequently, thin low-velocity layers can be difficult to identify, particularly when their thickness is below the resolution supported by the dominant wavelength, acquisition bandwidth, and available observations.

4.1 Deconvolution Cases

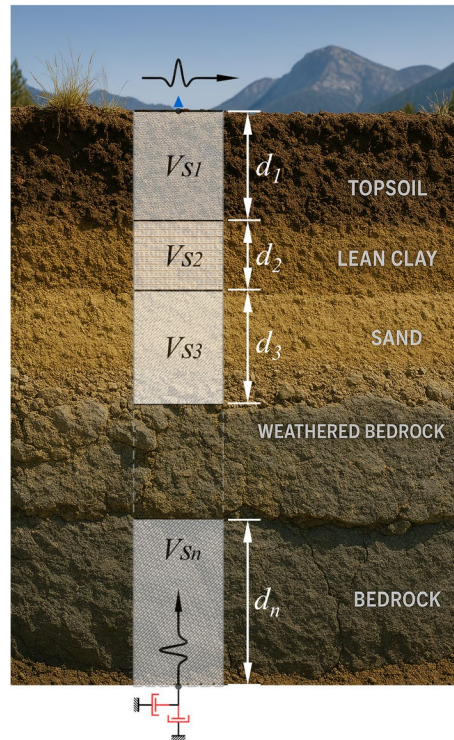


Figure 2. General layout of 1D layered soil model

Table 3. Material properties of the four-layer profile

Layer	Thickness (m)	V_s (m/s)	ζ (%)
1	26	287	3.313
2	66	480	2.449
3	107	719	1.851
4	81	1004	1.305

Table 4. Material properties of the six-layer profile

Layers	Thickness (m)	V_s (m/s)	ζ (%)
1	17	250	3.554
2	29	398	2.754
3	46	520	2.323
4	87	698 (418)	1.891 (2.673)
5	42	851	1.626
6	199	1085	1.177

The general configuration of the one-dimensional computational experiments is illustrated in Figure 2. The model represents vertically propagating shear waves through a horizontally layered soil column, with excitation applied at the base and the resulting motion recorded at the ground surface. Under these assumptions, the general vector wave equation is reduced to a scalar equation governing wave propagation through the soil column. Thirty subsurface velocity profiles were collected from published studies of sites across the United States, from which representative four-layer (4L) and six-layer (6L) profiles were developed for controlled numerical evaluation of the proposed framework under different levels of subsurface complexity. The thickness, shear-wave velocity, and damping ratio assigned to each layer of the 4L and 6L profiles are summarized in Tables 3 and 4, respectively. A constant mass density of 2,000 kg/m³ was assumed for all layers and computational cases. The 4L profile represents a relatively simple layered system, whereas the 6L profile provides a more detailed configuration for evaluating the framework under increased subsurface complexity. The values in parentheses for Layer 4 of the 6L profile indicate the alternative properties used for that layer in the corresponding computational case. As shown in Figure 2, a traction-free boundary condition is imposed at the ground surface, and an absorbing boundary condition is applied at the base to minimize artificial wave reflections. Each model is excited using a unit-amplitude Ricker wavelet with a central frequency of 3.5 Hz. The narrow-band pulse provides a controlled input for evaluating wave transmission, reflection, attenuation, and inversion performance within a frequency range relevant to geotechnical earthquake-engineering applications.

4.2 Site-Based Inversion Case Studies

The site-based inversion studies use the same general configurations as the preceding deconvolution cases, but the subsurface material properties are treated as unknown variables. For the elastic case, the target parameter is the shear-wave velocity profile. For the viscoelastic case, the shear-wave velocity and damping ratio are estimated simultaneously. To represent the limited availability of seismic measurements in engineering applications, only two recordings are used

as observational data for each inversion. The governing equations and the associated initial and boundary conditions provide additional constraints for estimating the unknown material properties.

4.3 Treasure Island Downhole Array (TIDA) Inversion

To evaluate the framework under field-informed conditions, we used seismic recordings from the TIDA (TIDA; CGS station 58642) for the magnitude 4.0 Alamo earthquake of September 5, 2008 (local time), together with the reference shear-wave velocity profile reported by Hallal and Cox (2023). The field records are available through the Center for Engineering Strong Motion Data and were processed from MATLAB files with a sampling interval of 0.005 s. Because the reference profile contains a pronounced velocity contrast at approximately 125 m, the inversion domain was restricted to the upper 80 m. Two signals were used: the measured surface recording and a model-derived response at a depth of 80 m, obtained using the 122-m downhole recording as input. The in-plane acceleration was calculated as $a_{\text{in}} = a_{\text{EW}} \sin 30^\circ - a_{\text{NS}} \cos 30^\circ$. The 6.0–11.0 s interval relative to the record start was extracted and filtered using a Butterworth band-pass filter with cutoff frequencies of 2 and 15 Hz and order parameter $N = 1$, applied forward and backward using SciPy's `filtfilt` for zero-phase filtering. The first 1,000 samples were tapered using a Tukey window with $\alpha = 0.04$, yielding a 5-s analysis segment with its time origin reset to zero. The shear-wave velocity profile within the target domain was then estimated by enforcing the governing partial differential equations and the corresponding initial and boundary conditions.

5. RESULTS AND DISCUSSION

5.1 Deconvolution Case Results

All deconvolution cases use a single neural network with four hidden layers and 100 neurons per layer. The known shear-wave velocity profile is incorporated into the training process through a prescribed spatial function. The only observation used for training is one surface recording with a sampling interval of 0.005 s.

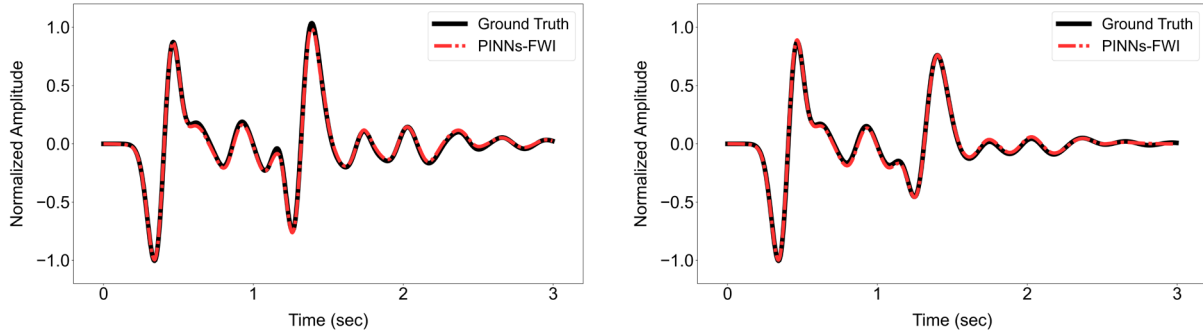


Figure 3. 4L elastic (left) and 4L viscoelastic (right) deconvolution results

Figures 3 and 4 compare the inferred base recordings with the corresponding reference inputs for the 4L and 6L profiles, respectively. The black traces represent the reference base excitation, while the red traces show the PINNs-FWI deconvolution results. In each case, the first pulse corresponds to the imposed unit-amplitude Ricker wavelet. The subsequent arrivals are associated with wave reflections and reverberations from the layer interfaces and the traction-free ground surface before returning to the base. In the viscoelastic cases, the amplitudes of these later arrivals are reduced because material damping accumulates over the propagation path. The 6L profile produces a more complex reflected waveform because it contains more interfaces and a longer propagation path. The longer path also increases attenuation in the viscoelastic case.

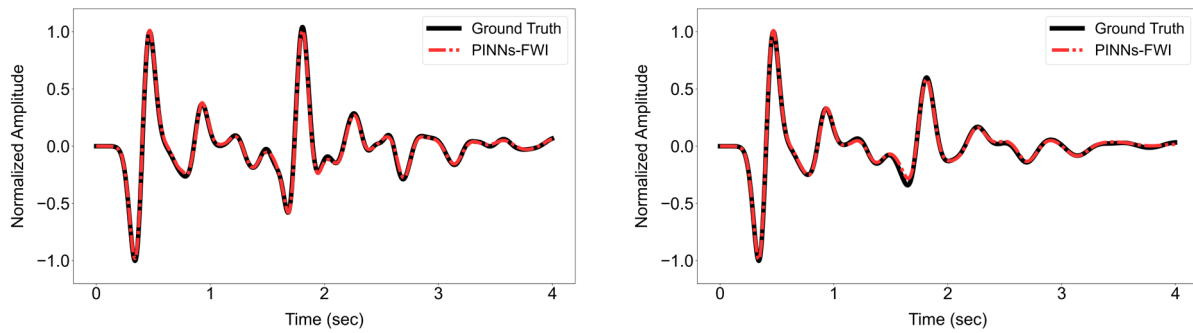


Figure 4. 6L elastic (left) and 6L viscoelastic (right) deconvolution results

The PINNs-FWI predictions reproduce the phase and normalized amplitude of the reference recordings in all four deconvolution cases. Waveforms are normalized by the peak surface-motion amplitude for visualization only. The normalized L_2 errors in Table 5 are calculated from the original signals. As summarized in Table 5, the normalized L_2 -norm errors are 4.91%,

4.95%, 5.22%, and 5.94% for the 4L elastic, 4L viscoelastic, 6L elastic, and 6L viscoelastic cases, respectively. The errors remain below 6%, although the 6L cases have slightly larger errors because of the additional interfaces and more complex reflected wavefield. These results show that the framework can recover the base motion from a single surface recording and represent the attenuation-related decay in the viscoelastic cases.

Table 5. Normalized errors for the base-motion deconvolution cases

Case	L ₂ norm error
4L elastic deconvolution	4.91%
4L viscoelastic deconvolution	4.95%
6L elastic deconvolution	5.22%
6L viscoelastic deconvolution	5.94%

5.2 Site-Based Inversion Results

For the 4L case, two neural networks are used: a forward network with four hidden layers and 80 neurons per layer (80×4), and an inverse network with four hidden layers and 20 neurons per layer (20×4). The networks are trained using the surface and base seismic recordings together with the governing equation and the corresponding initial and boundary conditions. Because the 6L profile contains more layer interfaces and produces a more complex wavefield, the forward-network capacity is increased to four hidden layers with 100 neurons per layer (100×4).

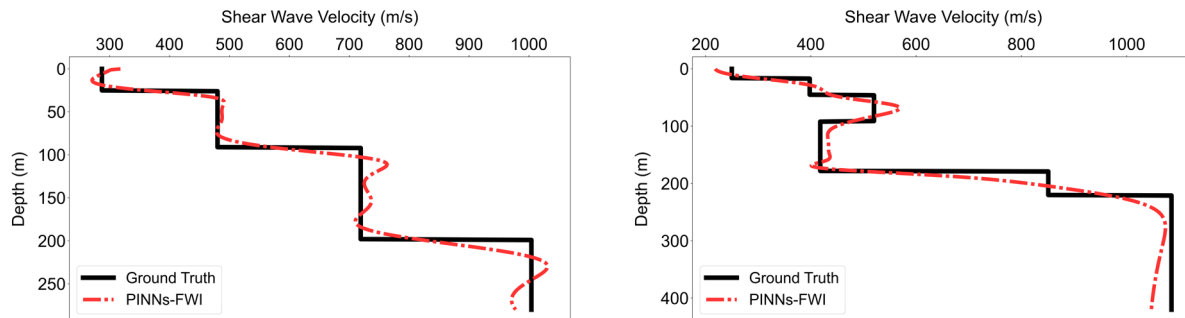


Figure 5. 4L elastic (left) and 6L elastic (right) case inversion results

Figure 5 presents the inverted shear-wave velocity profiles for the 4L and 6L cases. The corresponding mean absolute percentage errors (MAPEs) are 3.98% and 5.75%, respectively. For the 4L case, the inverted profile follows the reference values and identifies the main layer boundaries. The 6L inversion also recovers the general velocity trend, but it does not fully resolve the thin fifth layer, which has a higher shear-wave velocity than the overlying layer. The thickness of this layer is small relative to the resolution provided by the wavefield and available observations, causing the inversion to smooth its velocity contrast across the adjacent layers.

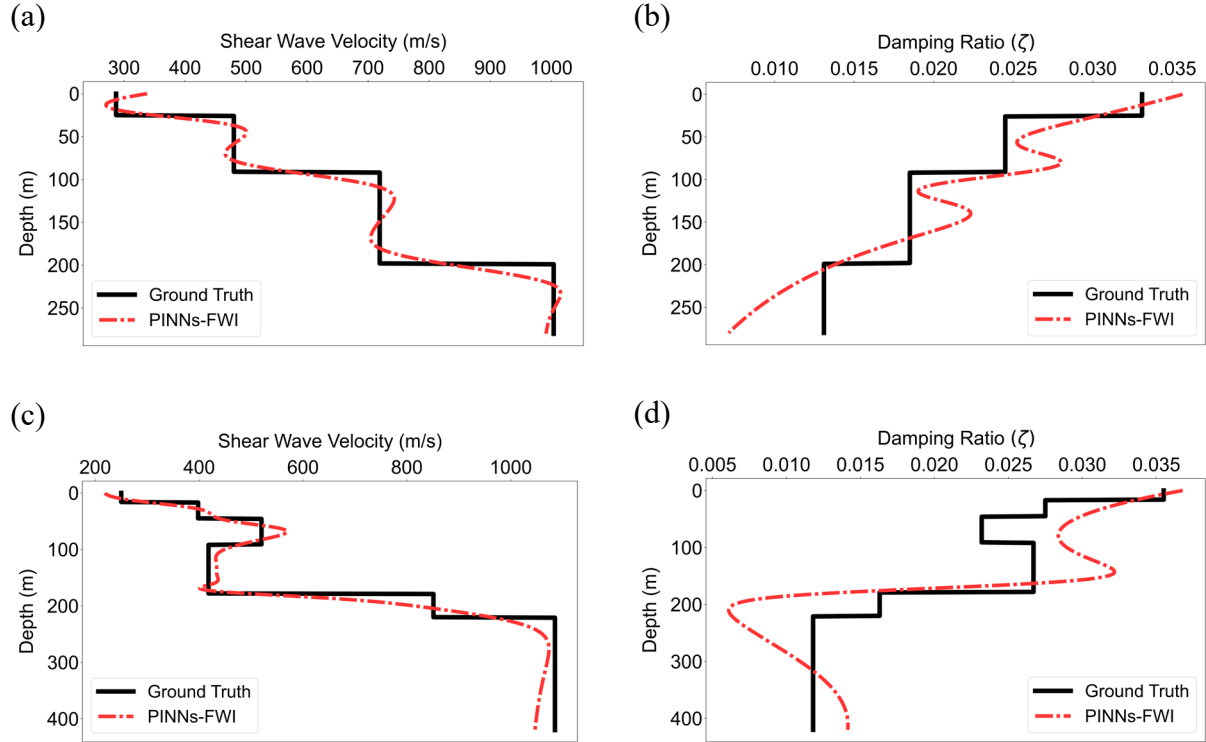


Figure 6. Joint shear-wave velocity and damping-ratio inversion results for the four- and six-layer viscoelastic cases

Figure 6 presents the viscoelastic inversion results for the 4L and 6L profiles, including the estimated shear-wave velocity and damping ratio. The velocity profiles follow the reference profiles, with MAPE values of 4.31% for the 4L case and 6.74% for the 6L case. In comparison, the damping-ratio estimates have larger errors of 17.93% and 29.82%, respectively, as reported in Table 6. The estimated damping profiles capture the general variation with depth but smooth the layer boundaries and deviate from the reference values in several layers. This lower accuracy is associated with the weaker sensitivity of the recorded wavefield to damping, the smaller numerical scale of the damping ratio, and the trade-off between velocity and damping during multiparameter inversion. The larger error for the 6L case reflects the additional layer interfaces and the increased difficulty of estimating both parameters from limited observations.

Table 6. Mean absolute percentage errors for the elastic and viscoelastic inversion cases

Case	V_s MAPE (%)	ζ MAPE (%)
4L elastic inversion	3.98%	-
4L viscoelastic inversion	4.31%	17.93%
6L elastic inversion	5.75%	-
6L viscoelastic inversion	6.74%	29.82%

5.3 TIDA Inversion Results

Because the TIDA field recordings contain more complex waveforms than the numerical cases, the capacity of both neural networks is increased. The forward network uses four hidden layers with 120 neurons per layer (120×4) to represent the spatial and temporal variations in the measured wavefield. The inverse network uses four hidden layers with 30 neurons per layer (30×4) to estimate the subsurface shear-wave velocity profile. The inversion uses a 5-s recording window with a sampling interval of 0.005 s. These network configurations provide the capacity required to model the field response and estimate velocity variations within the inversion domain.

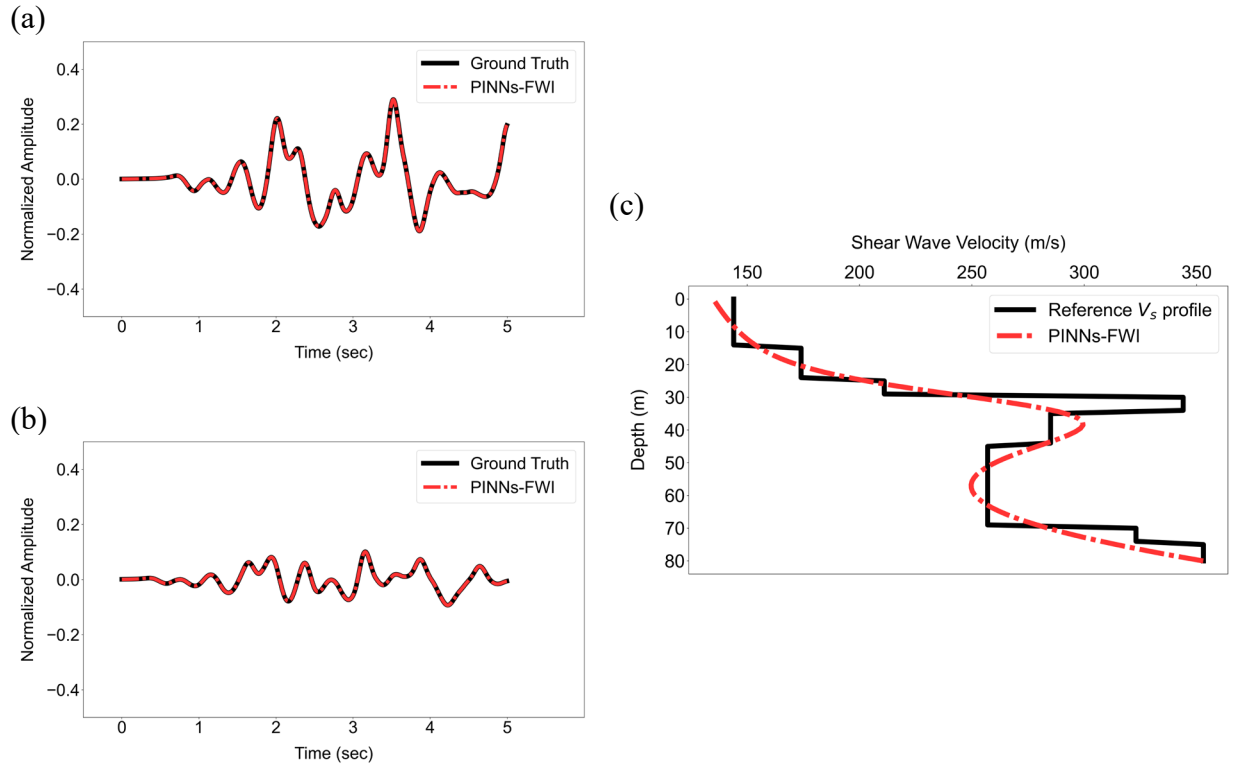


Figure 7. PINNs-FWI inversion results using TIDA recordings

Figure 7 presents the inversion results obtained using the TIDA field-informed case. Figures 7(a) and 7(b) compare the PINNs-FWI predictions with the measured surface recording and the model-derived response at a depth of 80 m, respectively. The predicted waveforms reproduce the arrival times, phases, and normalized amplitudes of the two target signals over the selected time interval. This agreement indicates that the inferred velocity model can reproduce the wave propagation represented by these constraints. However, because both signals are used during training, their agreement primarily demonstrates data fitting rather than independent validation.

Figure 7(c) compares the inverted shear-wave velocity profile with the reference profile. The inversion captures the overall increase in velocity with depth and identifies the principal velocity contrasts, with a shear-wave velocity MAPE of 9.62%. However, the estimated profile smooths several sharp layer boundaries and does not fully recover some thin layers, particularly between

depths of approximately 30 and 70 m. These differences can be attributed to the limited number of recordings, the frequency-dependent resolution of the wavefield, measurement noise, and the assumption of one-dimensional propagation. In addition, the field site may contain lateral heterogeneity and other effects that are not represented by the one-dimensional model. Despite these limitations, the results show that the PINNs-FWI framework can estimate the main features of a field shear-wave velocity profile from two wavefield constraints derived from the TIDA dataset.

6. CONCLUSION AND RECOMMENDATIONS

This study developed a PINNs-FWI framework for one-dimensional seismic-wave deconvolution and subsurface material characterization in elastic and viscoelastic layered media. The framework integrates seismic observations with the governing wave equations and associated initial and boundary conditions. For elastic inversion, the network estimates the shear-wave velocity profile, whereas the viscoelastic formulation simultaneously estimates shear-wave velocity and damping ratio using a Kelvin-Voigt material model.

The framework was evaluated using representative four- and six-layer profiles derived from published velocity profiles across the United States. The deconvolution results showed that the base motion could be recovered from a single surface recording, with normalized errors below 6% for all elastic and viscoelastic cases. The inversion results reproduced the overall velocity distributions and principal layer boundaries using only two recordings. However, the resolution decreased as the number of layers increased, particularly for thin layers and closely spaced interfaces.

The viscoelastic analyses demonstrated the feasibility of multiparameter inversion for velocity and damping. Shear-wave velocity was estimated more accurately than damping ratio because damping has a weaker influence on the recorded wavefield and differs substantially from velocity in numerical scale. Parameter trade-offs further reduced the accuracy of the damping estimates, especially for the six-layer profile.

The framework was also applied to field recordings from the Treasure Island Downhole Array. The inverted profile captured the general increase in shear-wave velocity with depth and the primary velocity contrasts within the upper 80 m. Differences from the reference profile were observed near thin layers and sharp boundaries because of limited measurements, noise, frequency-dependent resolution, and the assumption of one-dimensional propagation. Overall, the results demonstrate that PINNs-FWI can support seismic deconvolution and material-property estimation using limited observations, while its performance depends on subsurface complexity, parameter sensitivity, and data availability.

At its present stage, the framework should be viewed as a research tool rather than a replacement for established geophysical site-characterization methods. The results indicate that physics-informed inversion can complement conventional investigations when seismic observations are limited, particularly for estimating broad shear-wave velocity trends and identifying major stiffness contrasts. Additional field validation, uncertainty quantification, and extension to multidimensional conditions are needed before the method can be used for routine transportation-infrastructure screening or detection of localized subsurface anomalies.

Future work should focus on the following areas:

- Improve the resolution of thin layers and sharp interfaces through multiscale training and adaptive sampling.
- Reduce trade-offs between shear-wave velocity and damping ratio using parameter normalization, sensitivity-based weighting, or staged inversion.

- Validate the framework using additional field recordings and quantify uncertainty in the estimated material properties.
- Extend the formulation to two- and three-dimensional viscoelastic problems to account for lateral heterogeneity and complex wave propagation.

7. REFERENCES

- Abbas, A., J. P. Vantassel, B. R. Cox, K. Kumar, and J. Crocker. 2023. “A Frequency-Velocity CNN for Developing Near-Surface 2D Vs Images from Linear-Array, Active-Source Wavefield Measurements.” *Computers and Geotechnics* 156: 105305. <https://doi.org/10.1016/j.compgeo.2023.105305>.
- Adler, A., M. Araya-Polo, and T. Poggio. 2021. “Deep Learning for Seismic Inverse Problems: Toward the Acceleration of Geophysical Analysis Workflows.” *IEEE Signal Processing Magazine* 38 (2): 89–119. <https://doi.org/10.1109/MSP.2020.3037429>.
- Assimaki, D., L. F. Kallivokas, J. W. Kang, W. Li, and S. Kucukcoban. 2012. “Time-Domain Forward and Inverse Modeling of Lossy Soils with Frequency-Independent Q for Near-Surface Applications.” *Soil Dynamics and Earthquake Engineering* 43: 139–159. <https://doi.org/10.1016/j.soildyn.2012.07.001>.
- Baydin, A. G., B. A. Pearlmutter, A. A. Radul, and J. M. Siskind. 2018. “Automatic Differentiation in Machine Learning: A Survey.” *Journal of Machine Learning Research* 18 (153): 1–43. <https://jmlr.org/papers/v18/17-468.html>.
- Bunks, C., F. M. Saleck, S. Zaleski, and G. Chavent. 1995. “Multiscale Seismic Waveform Inversion.” *Geophysics* 60 (5): 1457–1473. <https://doi.org/10.1190/1.1443880>.
- Cao, W., Q. Li, J. Zhang, and W. Zhang. 2022. “Accelerating 2D and 3D Frequency-Domain Seismic Wave Modeling through Interpolating Frequency-Domain Wavefields by Deep Learning.” *Geophysics* 87 (4): T315–T328. <https://doi.org/10.1190/geo2021-0435.1>.
- Chai, X., T. Yang, H. Gu, G. Tang, W. Cao, and Y. Wang. 2023. “Geophysics-Steered Self-Supervised Learning for Deconvolution.” *Geophysical Journal International* 234 (1): 40–55. <https://doi.org/10.1093/gji/ggad015>.
- Cuomo, S., V. S. Di Cola, F. Giampaolo, G. Rozza, M. Raissi, and F. Piccialli. 2022. “Scientific Machine Learning through Physics-Informed Neural Networks: Where We Are and What’s Next.” *Journal of Scientific Computing* 92 (3): 88. <https://doi.org/10.1007/s10915-022-01939-z>.
- Ding, Y., S. Chen, H. Miyake, and X. Li. 2025. “Physics-Informed Neural Networks with Fourier Features for Seismic Wavefield Simulation in Time-Domain Nonsmooth Complex Media.” *IEEE Transactions on Geoscience and Remote Sensing* 63: 1–13. <https://doi.org/10.1109/TGRS.2025.3581638>.

- Ding, Y., S. Chen, X. Li, L. Jin, S. Luan, and H. Sun. 2023. "Physics-Constrained Neural Networks for Half-Space Seismic Wave Modeling." *Computers & Geosciences* 181: 105477. <https://doi.org/10.1016/j.cageo.2023.105477>.
- Fathi, A., L. F. Kallivokas, and B. Poursartip. 2015. "Full-Waveform Inversion in Three-Dimensional PML-Truncated Elastic Media." *Computer Methods in Applied Mechanics and Engineering* 296: 39–72. <https://doi.org/10.1016/j.cma.2015.07.008>.
- Gao, L., Y. Pan, A. Rieder, and T. Bohlen. 2021. "Multiparameter Viscoelastic Full-Waveform Inversion of Shallow-Seismic Surface Waves with a Pre-Conditioned Truncated Newton Method." *Geophysical Journal International* 227 (3): 2044–2057. <https://doi.org/10.1093/gji/ggab311>.
- Hallal, M. M., and B. R. Cox. 2023. "What Spatial Area Influences Seismic Site Response: Insights Gained from Multiazimuthal 2D Ground Response Analyses at the Treasure Island Downhole Array." *Journal of Geotechnical and Geoenvironmental Engineering* 149 (1): 04022124. <https://doi.org/10.1061/JGGEFK.GTENG-11023>.
- Kallivokas, L. F., A. Fathi, S. Kucukcoban, K. H. Stokoe, II, J. Bielak, and O. Ghattas. 2013. "Site Characterization Using Full Waveform Inversion." *Soil Dynamics and Earthquake Engineering* 47: 62–82. <https://doi.org/10.1016/j.soildyn.2012.12.012>.
- Kapoor, T., H. Wang, A. Núñez, and R. Dollevoet. 2024. "Physics-Informed Neural Networks for Solving Forward and Inverse Problems in Complex Beam Systems." *IEEE Transactions on Neural Networks and Learning Systems* 35 (5): 5981–5995. <https://doi.org/10.1109/TNNLS.2023.3310585>.
- Karimpouli, S., and P. Tahmasebi. 2020. "Physics Informed Machine Learning: Seismic Wave Equation." *Geoscience Frontiers* 11 (6): 1993–2001. <https://doi.org/10.1016/j.gsf.2020.07.007>.
- Karniadakis, G. E., I. G. Kevrekidis, L. Lu, P. Perdikaris, S. Wang, and L. Yang. 2021. "Physics-Informed Machine Learning." *Nature Reviews Physics* 3 (6): 422–440. <https://doi.org/10.1038/s42254-021-00314-5>.
- Kazei, V., O. Ovcharenko, P. Plotnitskii, D. Peter, X. Zhang, and T. Alkhalifah. 2021. "Mapping Full Seismic Waveforms to Vertical Velocity Profiles by Deep Learning." *Geophysics* 86 (5): R711–R721. <https://doi.org/10.1190/geo2019-0473.1>.
- Kingma, D. P., and J. Ba. 2014. "Adam: A Method for Stochastic Optimization." *arXiv*, 1412.6980. <https://doi.org/10.48550/arXiv.1412.6980>.
- Li, B., H. Wang, S. Feng, X. Yang, and Y. Lin. 2023. "Solving Seismic Wave Equations on Variable Velocity Models with Fourier Neural Operator." *IEEE Transactions on Geoscience and Remote Sensing* 61: 1–18. <https://doi.org/10.1109/TGRS.2023.3333663>.

- Lu, C., Y. Wang, X. Zou, J. Zong, and Q. Su. 2024. "Elastic Full-Waveform Inversion via Physics-Informed Recurrent Neural Network." *IEEE Transactions on Geoscience and Remote Sensing* 62: 1–16. <https://doi.org/10.1109/TGRS.2024.3450696>.
- Lysmer, J., and R. L. Kuhlemeyer. 1969. "Finite Dynamic Model for Infinite Media." *Journal of the Engineering Mechanics Division* 95 (4): 859–877. <https://doi.org/10.1061/JMCEA3.0001144>.
- Mohammadi, K., Y. Pu, and B. R. Cox. 2024. "Seismic Geotechnical Imaging Using Full-Waveform Inversion and Physics-Informed Neural Networks." *Japanese Geotechnical Society Special Publication* 10 (24): 889–895. <https://doi.org/10.3208/jgssp.v10.OS-13-03>.
- Moseley, B., A. Markham, and T. Nissen-Meyer. 2020a. "Solving the Wave Equation with Physics-Informed Deep Learning." *arXiv*, 2006.11894. <https://doi.org/10.48550/arXiv.2006.11894>.
- Moseley, B., T. Nissen-Meyer, and A. Markham. 2020b. "Deep Learning for Fast Simulation of Seismic Waves in Complex Media." *Solid Earth* 11 (4): 1527–1549. <https://doi.org/10.5194/se-11-1527-2020>.
- Operto, S., Y. Gholami, V. Prioux, et al. 2013. "A Guided Tour of Multiparameter Full-Waveform Inversion with Multicomponent Data: From Theory to Practice." *The Leading Edge* 32 (9): 1040–1054. <https://doi.org/10.1190/tle32091040.1>.
- Park, C. B., R. D. Miller, and J. Xia. 1999. "Multichannel Analysis of Surface Waves." *Geophysics* 64 (3): 800–808. <https://doi.org/10.1190/1.1444590>.
- Phoon, K.-K., and F. H. Kulhawy. 1999. "Characterization of Geotechnical Variability." *Canadian Geotechnical Journal* 36 (4): 612–624. <https://doi.org/10.1139/t99-038>.
- Poursartip, B., A. Fathi, and J. L. Tassoulas. 2020. "Large-Scale Simulation of Seismic Wave Motion: A Review." *Soil Dynamics and Earthquake Engineering* 129: 105909. <https://doi.org/10.1016/j.soildyn.2019.105909>.
- Prioux, V., R. Brossier, S. Operto, and J. Virieux. 2013. "Multiparameter Full Waveform Inversion of Multicomponent Ocean-Bottom-Cable Data from the Valhall Field. Part 2: Imaging Compressive-Wave and Shear-Wave Velocities." *Geophysical Journal International* 194 (3): 1665–1681. <https://doi.org/10.1093/gji/ggt178>.
- Qu, S., E. Verschuur, and Y. Chen. 2019. "Full-Waveform Inversion and Joint Migration Inversion with an Automatic Directional Total Variation Constraint." *Geophysics* 84 (2): R175–R183. <https://doi.org/10.1190/geo2018-0085.1>.
- Raissi, M., P. Perdikaris, and G. E. Karniadakis. 2019. "Physics-Informed Neural Networks: A Deep Learning Framework for Solving Forward and Inverse Problems Involving Nonlinear Partial Differential Equations." *Journal of Computational Physics* 378: 686–707. <https://doi.org/10.1016/j.jcp.2018.10.045>.

- Rasht-Behesht, M., C. Huber, K. Shukla, and G. E. Karniadakis. 2022. “Physics-Informed Neural Networks (PINNs) for Wave Propagation and Full Waveform Inversions.” *Journal of Geophysical Research: Solid Earth* 127 (5): e2021JB023120. <https://doi.org/10.1029/2021JB023120>.
- Ren, P., C. Rao, S. Chen, J.-X. Wang, H. Sun, and Y. Liu. 2024. “SeismicNet: Physics-Informed Neural Networks for Seismic Wave Modeling in Semi-Infinite Domain.” *Computer Physics Communications* 295: 109010. <https://doi.org/10.1016/j.cpc.2023.109010>.
- Ren, P., C. Rao, H. Sun, and Y. Liu. 2023. “Physics-Informed Neural Network for Seismic Wave Inversion in Layered Semi-Infinite Domain.” *arXiv*, 2305.05150. <https://doi.org/10.48550/arXiv.2305.05150>.
- Schweiger, H. F., and G. M. Peschl. 2005. “Reliability Analysis in Geotechnics with the Random Set Finite Element Method.” *Computers and Geotechnics* 32 (6): 422–435. <https://doi.org/10.1016/j.compgeo.2005.07.002>.
- Shukla, K., A. D. Jagtap, J. L. Blackshire, D. Sparkman, and G. E. Karniadakis. 2022. “A Physics-Informed Neural Network for Quantifying the Microstructural Properties of Polycrystalline Nickel Using Ultrasound Data: A Promising Approach for Solving Inverse Problems.” *IEEE Signal Processing Magazine* 39 (1): 68–77. <https://doi.org/10.1109/MSP.2021.3118904>.
- Socco, L. V., S. Foti, and D. Boiero. 2010. “Surface-Wave Analysis for Building Near-Surface Velocity Models — Established Approaches and New Perspectives.” *Geophysics* 75 (5): 75A83–75A102. <https://doi.org/10.1190/1.3479491>.
- Song, C., and Y. Wang. 2023. “Simulating Seismic Multifrequency Wavefields with the Fourier Feature Physics-Informed Neural Network.” *Geophysical Journal International* 232 (3): 1503–1514. <https://doi.org/10.1093/gji/ggac399>.
- Stokoe, K. H., II, S. G. Wright, J. A. Bay, and J. M. Roesset. 1994. “Characterization of Geotechnical Sites by SASW Method.” In *Geophysical Characterization of Sites*, edited by R. D. Woods, 15–25. Oxford & IBH Publishing.
- Tarantola, A. 1984. “Inversion of Seismic Reflection Data in the Acoustic Approximation.” *Geophysics* 49 (8): 1259–1266. <https://doi.org/10.1190/1.1441754>.
- Teague, D. P., and B. R. Cox. 2016. “Site Response Implications Associated with Using Non-Unique Vs Profiles from Surface Wave Inversion in Comparison with Other Commonly Used Methods of Accounting for Vs Uncertainty.” *Soil Dynamics and Earthquake Engineering* 91: 87–103. <https://doi.org/10.1016/j.soildyn.2016.07.028>.
- Vantassel, J. P., K. Kumar, and B. R. Cox. 2022. “Using Convolutional Neural Networks to Develop Starting Models for Near-Surface 2-D Full Waveform Inversion.” *Geophysical Journal International* 231 (1): 72–90. <https://doi.org/10.1093/gji/ggac179>.

- Virieux, J., A. Asnaashari, R. Brossier, L. Métivier, A. Ribodetti, and W. Zhou. 2014. “An Introduction to Full Waveform Inversion.” In *Encyclopedia of Exploration Geophysics*, edited by V. Grechka and K. Wapenaar, R1-1–R1-40. Society of Exploration Geophysicists. <https://doi.org/10.1190/1.9781560803027.entry6>.
- Virieux, J., and S. Operto. 2009. “An Overview of Full-Waveform Inversion in Exploration Geophysics.” *Geophysics* 74 (6): WCC1–WCC26. <https://doi.org/10.1190/1.3238367>.
- Wu, X., J. Ma, X. Si, et al. 2023. “Sensing Prior Constraints in Deep Neural Networks for Solving Exploration Geophysical Problems.” *Proceedings of the National Academy of Sciences* 120 (23): e2219573120. <https://doi.org/10.1073/pnas.2219573120>.
- Wu, Y., W. Wang, Y. Wang, and G. A. McMechan. 2025. “Deep Learning-Based Self-Supervised Multi-Parameter Inversion.” *Geophysical Journal International* 243 (2): ggaf332. <https://doi.org/10.1093/gji/ggaf332>.
- Yang, F., and J. Ma. 2023. “FWIGAN: Full-Waveform Inversion via a Physics-Informed Generative Adversarial Network.” *Journal of Geophysical Research: Solid Earth* 128 (4): e2022JB025493. <https://doi.org/10.1029/2022JB025493>.
- Zhang, Y., X. Zhu, and J. Gao. 2023. “Seismic Inversion Based on Acoustic Wave Equations Using Physics-Informed Neural Network.” *IEEE Transactions on Geoscience and Remote Sensing* 61: 1–11. <https://doi.org/10.1109/TGRS.2023.3236973>.
- Zhang, Z., T. Alkhalifah, and Y. Liu. 2024. “Full Dispersion-Spectrum Inversion of Surface Waves.” *Journal of Geophysical Research: Solid Earth* 129 (6): e2023JB028469. <https://doi.org/10.1029/2023JB028469>.
- Zhu, Z., Z. Wang, Y. Feng, and W. Zheng. 2025. “Solution and Application of Two-Dimensional Seismic Wavefield Evolution Based on Physics-Informed Neural Networks.” *Engineering Applications of Artificial Intelligence* 150: 110652. <https://doi.org/10.1016/j.engappai.2025.110652>.
- Zou, J., C. Liu, Y. Wang, C. Song, U. B. Waheed, and P. Zhao. 2025. “Accelerating the Convergence of Physics-Informed Neural Networks for Seismic Wave Simulation.” *Geophysics* 90 (2): T23–T32. <https://doi.org/10.1190/geo2024-0264.1>.



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