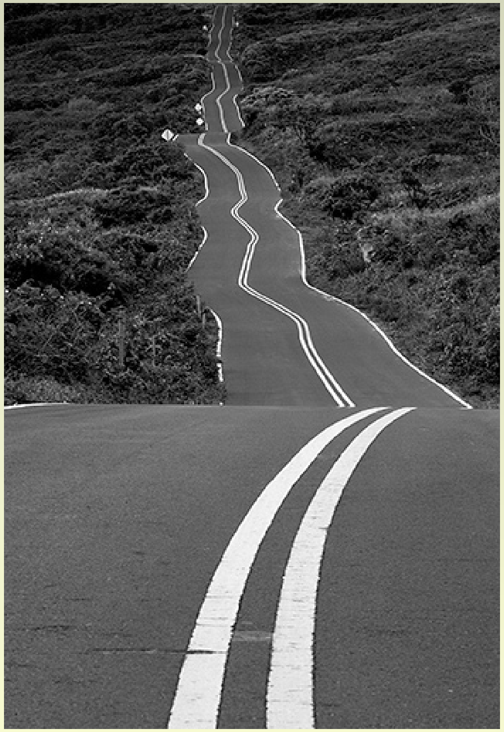
DESIGN OF VERTICAL CURVES



Minh Huynh

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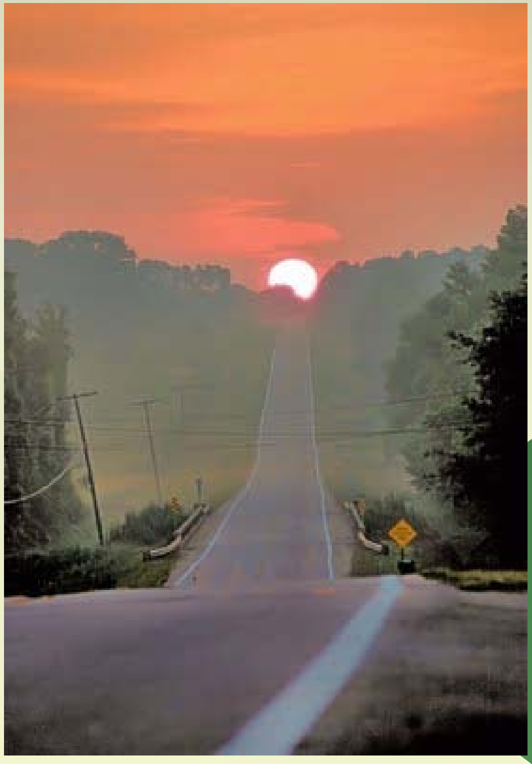
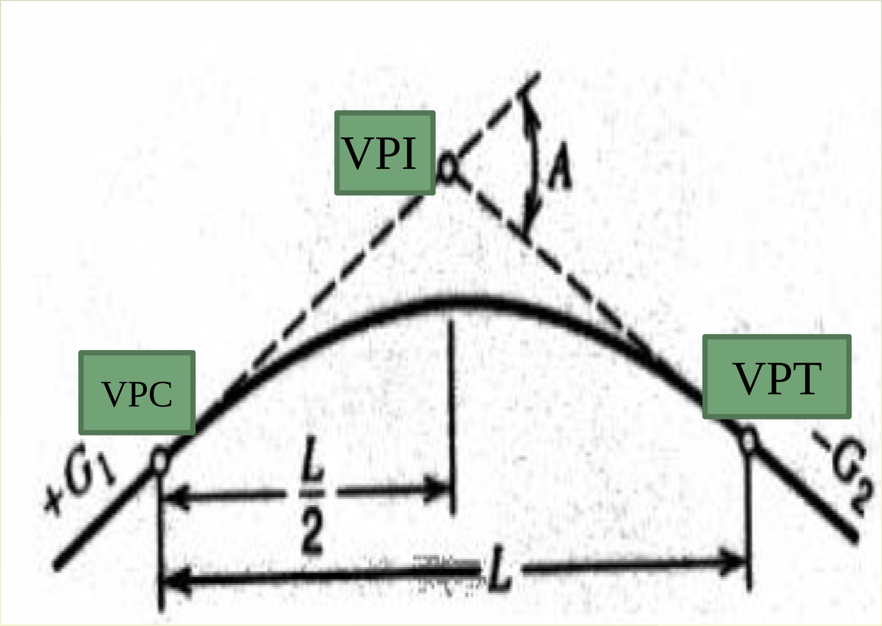


What is a vertical curve?

- A parabolic curve that is applied to make a smooth and safe transition between two grades on a roadway or a highway.

VPC: Vertical Point of Curvature
VPI: Vertical Point of Intersection
VPT: Vertical Point of Tangency
G1, G2: Tangent grades in percent
A: Algebraic difference in grades
L: Length of vertical curve



When are vertical curves applied?

- At an intersection of two slopes on a highway or a roadway
- To provide a safe and comfort ride for vehicles on a roadway.

Types of Vertical Curves

- Two kinds of vertical curve
  - Crest Vertical Curves
    - + Type I and Type II
  - Sag Vertical Curves
    - + Type III and Type IV.

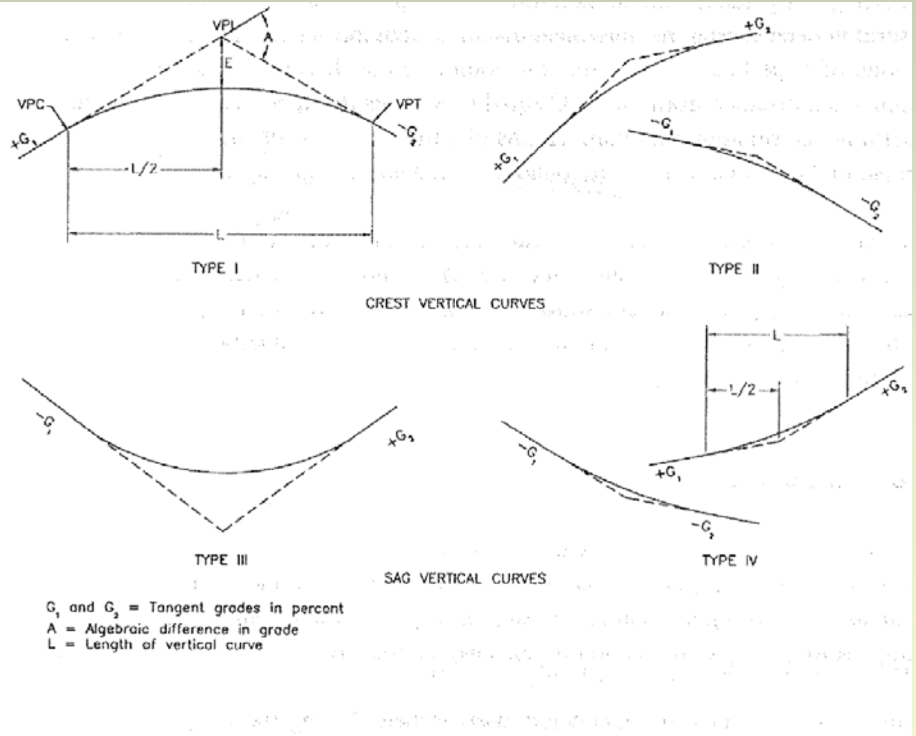


Exhibit 3-73. Types of Vertical Curves

K values

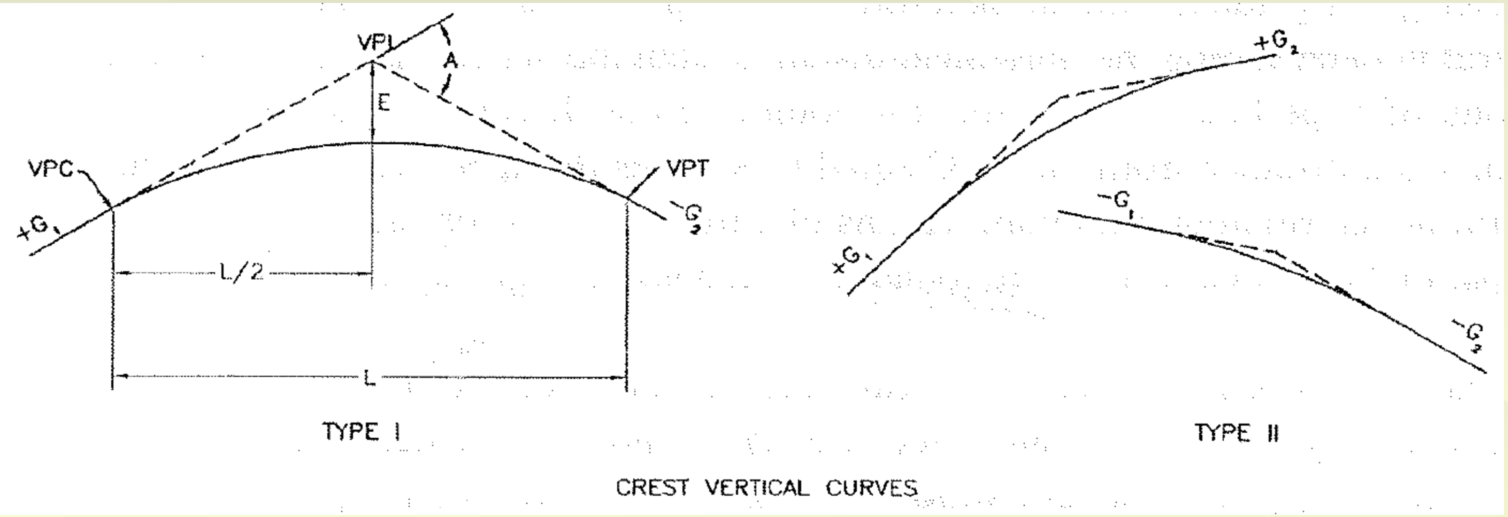
- Def: the horizontal distance in feet (meters) needed to make 1% change in gradient.

$$K = \frac{L}{A}$$

- Application:
  - To determine the minimum lengths of vertical curves
  - To determine the horizontal distance from the VPC to the high point of Type I or the low point of Type III

Crest Vertical Curves

- Minimum length of a crest vertical curve needs to satisfy the safety, comfort, and appearance criteria.
- Minimum length of a crest vertical curve is equal 3 time the design speed (only for English Unit).



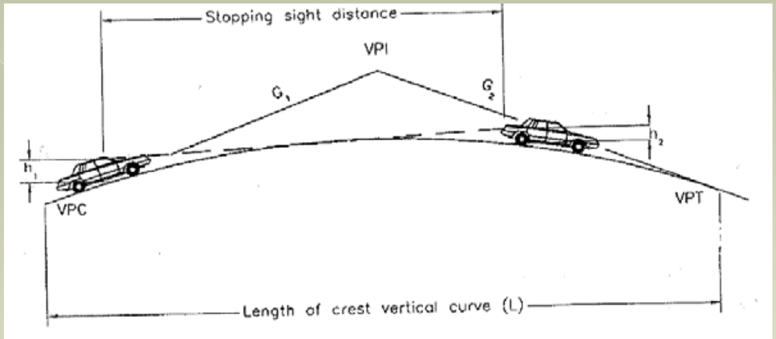
- General equation for the length of a crest vertical curve in terms of algebraic difference in grades.
  - When S is less than L
$$L = \frac{AS^2}{100\left(\sqrt{2h_1} + \sqrt{2h_2}\right)^2}$$
  - When S is greater than L
$$L = 2S - \frac{200\left(\sqrt{h_1} + \sqrt{h_2}\right)^2}{A}$$

L: length of vertical curve, ft
S: sight distance, ft
A: algebraic difference in grades, percent
h_1: height of eye above roadway, ft (3.5ft)
h2: height of object above roadway surface, ft (2ft)

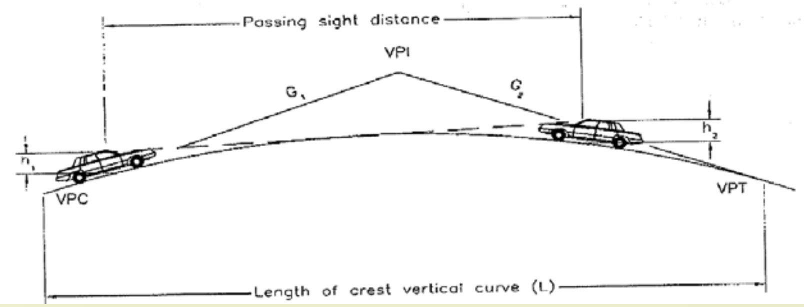
- These equations are often used to check the design speed of an existing vertical curve. K values are preferred to be used when design a new vertical curve because it provides a better safety distance.

Design controls

- Design base on stopping sight distance



- Design base on passing sigh distance



- MUTCD passing sight distance
- Decision sight distance

Stopping Sight Distance

- Def: the total distances from when the driver decides to apply the break until the vehicle stop.

Metric					US Customary				
			Stopping sight distance					Stopping sight distance	
Design speed (km/h)	Brake reaction distance (m)	Braking distance on level (m)	Calculated (m)	Design (m)	Design speed (mph)	Brake reaction distance (ft)	Braking distance on level (ft)	Calculated (ft)	Design (ft)
20	13.9	4.6	18.5	20	15	55.1	21.6	76.7	80
30	20.9	10.3	31.2	35	20	73.5	38.4	111.9	115
40	27.8	18.4	46.2	50	25	91.9	60.0	151.9	155
50	34.8	28.7	63.5	65	30	110.3	86.4	196.7	200
60	41.7	41.3	83.0	85	35	128.6	117.6	246.2	250
70	48.7	56.2	104.9	105	40	147.0	153.6	300.6	305
80	55.6	73.4	129.0	130	45	165.4	194.4	359.8	360
90	62.6	92.9	155.5	160	50	183.8	240.0	423.8	425
100	69.5	114.7	184.2	185	55	202.1	290.3	492.4	495
110	76.5	138.8	215.3	220	60	220.5	345.5	566.0	570
120	83.4	165.2	248.6	250	65	238.9	405.5	644.4	645
130	90.4	193.8	284.2	285	70	257.3	470.3	727.6	730
					75	275.6	539.9	815.5	820
					80	294.0	614.3	908.3	910

Note: Brake reaction distance predicated on a time of 2.5 s; deceleration rate of 3.4 m/s^2 [11.2 ft/s^2] used to determine calculated sight distance.

Exhibit 3-1. Stopping Sight Distance

$$d = 1.47Vt + 1.075V^2/a$$

t: break reaction time, (assumed 2.5s)
V: design speed, mph
a: deceleration rate, ft/s^2

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Metric				US Customary			
Design speed (km/h)	Stopping sight distance (m)	Rate of vertical curvature, K^a		Design speed (mph)	Stopping sight distance (ft)	Rate of vertical curvature, K^a	
		Calculated	Design			Calculated	Design
20	20	0.6	1	15	80	3.0	3
30	35	1.9	2	20	115	6.1	7
40	50	3.8	4	25	155	11.1	12
50	65	6.4	7	30	200	18.5	19
60	85	11.0	11	35	250	29.0	29
70	105	16.8	17	40	305	43.1	44
80	130	25.7	26	45	360	60.1	61
90	160	38.9	39	50	425	83.7	84
100	185	52.0	52	55	495	113.5	114
110	220	73.6	74	60	570	150.6	151
120	250	95.0	95	65	645	192.8	193
130	285	123.4	124	70	730	246.9	247
				75	820	311.6	312
				80	910	383.7	384

[a] Rate of vertical curvature, K, is the length of curve per percent algebraic difference in intersecting grades (A). $K = L/A$

Exhibit 3-72. Design Controls for Stopping Sight Distance and for Crest Vertical Curves

- In Exhibit 3-72, K values are calculated by the equation.
- K values can also be used when S > L because there is no significant error between S>L and S<L.

$$K = \frac{S^2}{2158}$$

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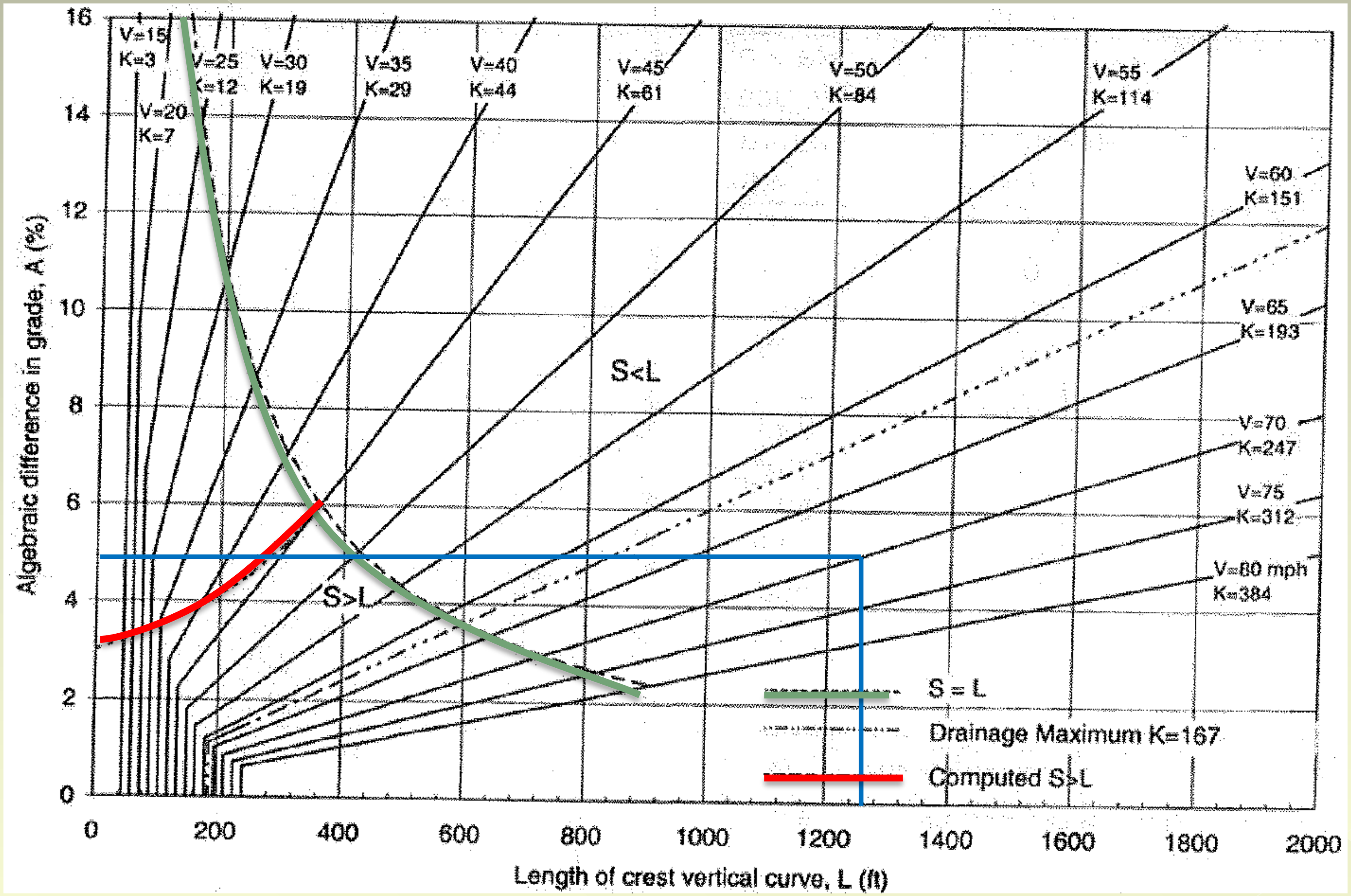
Exhibit 3-71



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Example

An engineer is assigned to design a vertical curve for a highway with the design speed is 70 mph. Knowing that the gradients are 3% uphill and -2% downhill. What is the minimum design length of the vertical curve?

<u>Solution:</u>

- Find the value of K from exhibit 3-72. (page 272 of AASHTO 2004)

  For 70mph K = 247

- Find the value of algebraic difference in grade

  A= G1 - G2

  A= 3 - (-2)

  A= 5

- Find minimum length of the vertical curve by using equation L= K*A

  L = 247 * 5 = 1235 ft

$$K = \frac{L}{A}$$

Or using exhibit 3-71 (page 271 of AASHTO)

Passing sight distance

- Def: the distance that allows a driver to complete a normal pass while that driver can observe that there is no potential threat ahead before making the pass.
- Total of below distances is the design distance for two lanes highway.
  - Initial maneuver distance
  - Distance while passing vehicle occupies left lane
  - Clearance distance
  - Distance traversed by an opposing vehicle
- Rarely use in crest vertical design because it is difficult to fit the length of the curve.
- Can be used when the design speed is low and does not have high gradient, or higher speed with very small algebraic difference in grades
- Commonly use at location where combinations of alignment and profile do not need the use of crest vertical curves
- Height of an object is assumed to be 3.5ft instead of 2ft in general equation. (Simplified equation can be found on page 270 of AASHTO Green Book)

Metric					US Customary				
Design speed (km/h)	Assumed speeds (km/h)		Passing sight distance (m)		Design speed (mph)	Assumed speeds (mph)		Passing sight distance (ft)	
	Passed vehicle	Passing vehicle	From Exhibit 3-6	Rounded for design		Passed vehicle	Passing vehicle	From Exhibit 3-6	Rounded for design
30	29	44	200	200	20	18	28	706	710
40	36	51	266	270	25	22	32	897	900
50	44	59	341	345	30	26	36	1088	1090
60	51	66	407	410	35	30	40	1279	1280
70	59	74	482	485	40	34	44	1470	1470
80	65	80	538	540	45	37	47	1625	1625
90	73	88	613	615	50	41	51	1832	1835
100	79	94	670	670	55	44	54	1984	1985
110	85	100	727	730	60	47	57	2133	2135
120	90	105	774	775	65	50	60	2281	2285
130	94	109	812	815	70	54	64	2479	2480
					75	56	66	2578	2580
					80	58	68	2677	2680

Exhibit 3-7. Passing Sight Distance for Design of Two-Lane Highways

In exhibit 3-7, K values are calculated by the equation
Passing vehicle speed usually is assumed 10 to 15 mph higher than the passed vehicle.

$$K = \frac{S^2}{2800}$$

From page 124 of AASHTO's *A Policy on Geometric Design of Highways and Streets 2004*

Metric			US Customary		
Design speed (km/h)	Passing sight distance (m)	Rate of vertical curvature, K* design	Design speed (mph)	Passing sight distance (ft)	Rate of vertical curvature, K* design
30	200	46	20	710	180
40	270	84	25	900	289
50	345	138	30	1090	424
60	410	195	35	1280	585
70	485	272	40	1470	772
80	540	338	45	1625	943
90	615	438	50	1835	1203
100	670	520	55	1985	1407
110	730	617	60	2135	1628
120	775	695	65	2285	1865
130	815	769	70	2480	2197
			75	2580	2377
			80	2680	2565

Note: *Rate of vertical curvature, K, is the length of curve per percent algebraic difference in intersecting grades (A). $K = L/A$

Exhibit 3-73. Design Controls for Crest Vertical Curves Based on Passing Sight Distance

These length are 7 to 10 times longer than the stopping sight distances.

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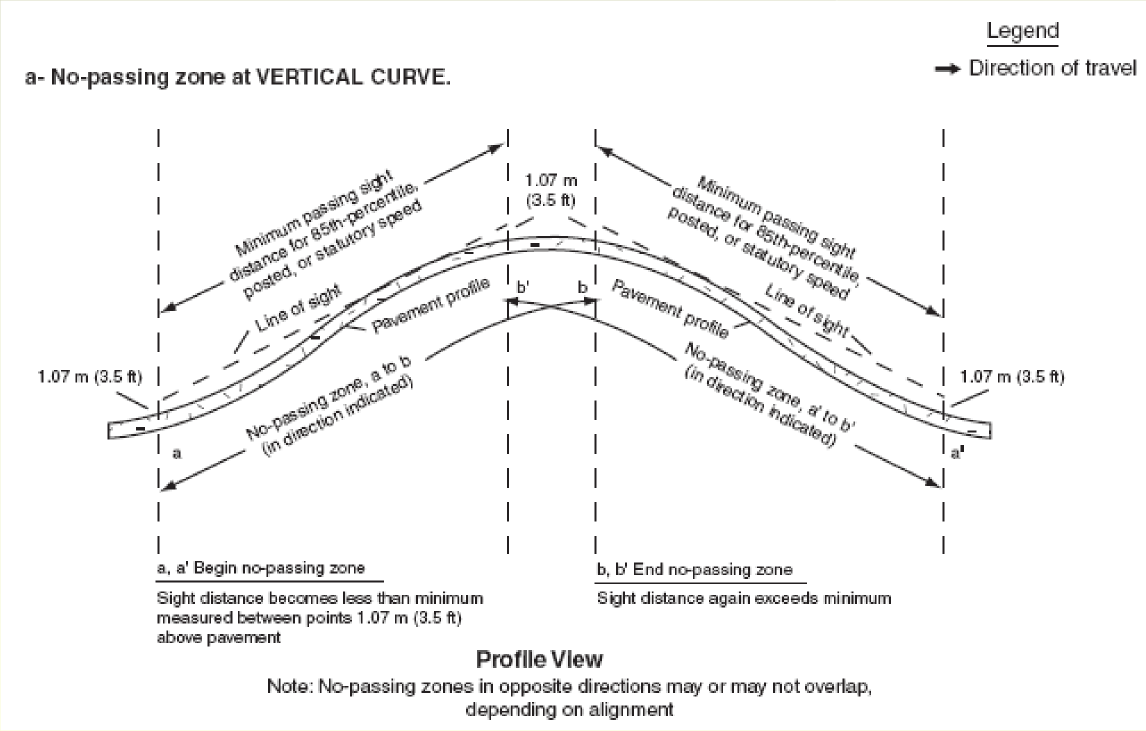
MUTCD Passing Sight Distance

- Def: "the distance at which an object 3.5ft above pavement surface can be seen from a point 3.5ft above the pavement." (page 3B-5)
- Only use for traffic operation-control needs, such as placing "No Passing" zone warrant.
- Minimum passing sight distances are shown in Table 3B-1, page 3B-7 of 2003 MUTCD book.



Page 3B-8 of MUTCD 2003 version

Table 3B-1. Minimum Passing Sight Distances

85th-Percentile or Posted or Statutory Speed Limit (km/h)	Minimum Passing Sight Distance (meters)
40	140
50	160
60	180
70	210
80	245
90	280
100	320
110	355
120	395

85th-Percentile or Posted or Statutory Speed Limit (mph)	Minimum Passing Sight Distance (feet)
25	450
30	500
35	550
40	600
45	700
50	800
55	900
60	1,000
65	1,100
70	1,200

Page 3B-7 of MUTCD 2003 version

Decision sight distance

- Def: “the distance needed for a driver to detect an unexpected or otherwise difficult-to-perceive information sources or condition in a roadway environment that may be visually cluttered, recognize the condition or its potential threat, select an appropriate speed and path, and initiate and complete the maneuver safety and efficiently.” (page 115, AASHTO)

- Exhibit 3-3 provides values for the decision sight distances that may be appropriate at critical locations and serve as criteria in evaluation the suitability of the available sight distance.

Metric					
Design speed (km/h)	Decision sight distance (m)				
	Avoidance maneuver				
	A	B	C	D	E
50	70	155	145	170	195
60	95	195	170	205	235
70	115	235	200	235	275
80	140	280	230	270	315
90	170	325	270	315	360
100	200	370	315	355	400
110	235	420	330	380	430
120	265	470	360	415	470
130	305	525	390	450	510

US Customary					
Design speed (mph)	Decision sight distance (ft)				
	Avoidance maneuver				
	A	B	C	D	E
30	220	490	450	535	620
35	275	590	525	625	720
40	330	690	600	715	825
45	395	800	675	800	930
50	465	910	750	890	1030
55	535	1030	865	980	1135
60	610	1150	990	1125	1280
65	695	1275	1050	1220	1365
70	780	1410	1105	1275	1445
75	875	1545	1180	1365	1545
80	970	1685	1260	1455	1650

Avoidance Maneuver A: Stop on rural road—t = 3.0 s
Avoidance Maneuver B: Stop on urban road—t = 9.1 s
Avoidance Maneuver C: Speed/path/direction change on rural road—t varies between 10.2 and 11.2 s
Avoidance Maneuver D: Speed/path/direction change on suburban road—t varies between 12.1 and 12.9 s
Avoidance Maneuver E: Speed/path/direction change on urban road—t varies between 14.0 and 14.5 s

Exhibit 3-3. Decision Sight Distance

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- Avoidance maneuvers A and B are determined as

$$d = 1.47Vt + \frac{1.075V^2}{a}$$

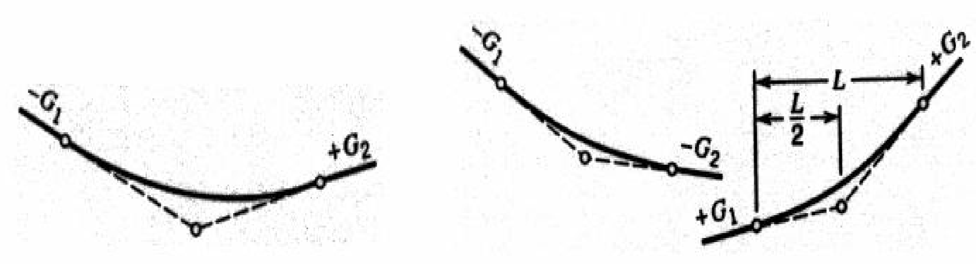
- Avoidance maneuvers C, D, and E are determined as

$$d = 1.47Vt$$

  - d: decision distance
  - t: pre-maneuver time, s
  - V: design speed, mph
  - A: driver acceleration, ft/s^2

Sag Vertical curves

- A design of a sag vertical curves need to satisfy at least four difference criteria.
  - Head light sight distance
  - Passenger comfort
  - Drainage control
  - General appearance



Sag Vertical Curves

- General equation

S < L

$$L = \frac{AS^2}{200(h_1 + S\tan\beta)}$$

S>L

$$L = 2S - \frac{200(h_1 + S\tan\beta)}{A}$$

L: length of sag vertical cure, ft

S: light beam distance, ft

A: algebraic difference in grades, percent

β: angle of light beam intersects the surface of the roadway, degree (assumed 1^o)

h_1: head light height, (assumed 2ft)

Design considerations

- The design length of a sag vertical curve is based on the head light sight distance, but the head light sight distance needs to be designed almost equal to the stopping sight distance because of safety criterion. Therefore, stopping sight distance values can be use for S value in general equation. Therefore, K values can be used to calculate the length of the curve.
- For passenger comfort, the below equation can be used.
  - L: length of sag vertical curve, ft
  - A: algebraic difference in grades, percent
  - V: design speed, mph

$$L = \frac{AV^2}{46.5}$$

- Drainage of curbed roadways needs to retain a grade at least 0.5 percent or sometimes 0.3 percent for outer edges of the roadway.
- For appearance, the minimum curve length can be calculated by equation L=100A for small or intermediate values of A.

Metric				US Customary			
Design speed (km/h)	Stopping sight distance (m)	Rate of vertical curvature, K^a		Design speed (mph)	Stopping sight distance (ft)	Rate of vertical curvature, K^a	
		Calculated	Design			Calculated	Design
20	20	2.1	3	15	80	9.4	10
30	35	5.1	6	20	115	16.5	17
40	50	8.5	9	25	155	25.5	26
50	65	12.2	13	30	200	36.4	37
60	85	17.3	18	35	250	49.0	49
70	105	22.6	23	40	305	63.4	64
80	130	29.4	30	45	360	78.1	79
90	160	37.6	38	50	425	95.7	96
100	185	44.6	45	55	495	114.9	115
110	220	54.4	55	60	570	135.7	136
120	250	62.8	63	65	645	156.5	157
130	285	72.7	73	70	730	180.3	181
				75	820	205.6	206
				80	910	231.0	231

[a] Rate of vertical curvature, K, is the length of curve (m) per percent algebraic difference intersecting grades (A). $K = L/A$

Exhibit 3-75. Design Controls for Sag Vertical Curves

In exhibit 3-75, K values are calculated by equation

$$K = \frac{S^2}{400 + 3.5S}$$

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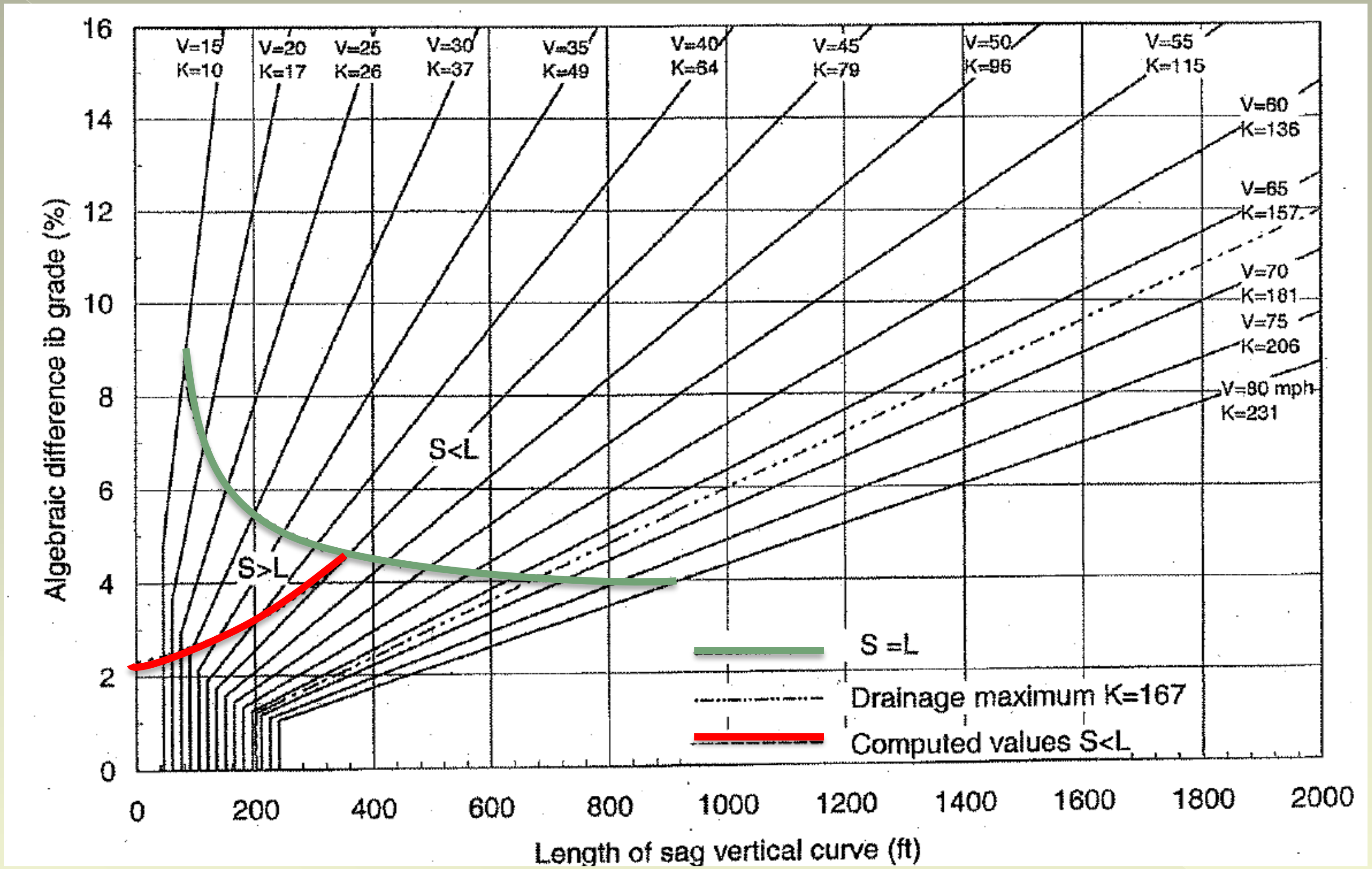

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Other Design considerations for Vertical Curve Designs

- Paying more attention to the drainage design when value of K >167
- The length of vertical curve can be computed by using K values in both crest and sag vertical curves.
- Minimum length of a crest vertical curve is equal 3 time the design speed (only for English Unit).
- The “roller-coaster” or the “hidden up” type of profile should be avoided.
- Two vertical curves in the same direction separated by a short section of tangent grade should be avoided.
- On long grades, the steepest grades should be placed at the bottom of the curve and flatten the grades near the top of ascent.
- It is desirable to reduce the grade through the intersection where at-grade intersection occur on roadway sections with moderate to steep grades.
- Sag vertical curves should be avoided in cuts unless adequate drainage can be provided.
- The stopping sight distance for trucks is not necessary to be considered in designing vertical because the truck driver able to see farther than passenger car. For that reason, the stopping sight distance for trucks and passenger cars is balance.
- Most of cases the stopping sight distance will be used for vertical design length, but engineering judgments also get involve in decision making.

Sources

- American Association of State Highway and Transportation Officials (AASHTO). (2004). *A Policy on Geometric Design of Highways and Streets*, Fifth Edition. Washington, D.C
- Manual on Uniform Traffic Control Devices (MUTCD). (2003). Millennium Edition.